On International Cooperation in Monetary and Fiscal Policy

Master's Thesis

For the Attainment of the Degree
Master's of Arts in Economics and Management Sciences

From the School of Business and Economics
Humboldt University at Berlin

Submitted by

Alexander Warren Zieroth

(Matriculation N° 184987)

To **Prof. Harald Uhlig, Ph.D.**
(Primary Examiner)

In Berlin, Germany
On August 5, 2005

Abstract

I develop a two-country dynamic stochastic general equilibrium model featuring monopolistic competition and staggered price setting with an interest-rate setting rule and distortionary taxes in each country to examine the potential for policy cooperation. I show that a tax on consumption in this framework is very powerful, distorting consumption risk sharing via the real exchange rate. Monetary policy is specified as a simple Taylor Rule common with a closed economy model. Endogenous fiscal policy is effected by means of lagged feedback rules, and two alternatives are compared. Using technology as feedback is not found to be welfare improving compared with random walk taxation and cooperation is to be recommended to overcome mutually disadvantageous unilateral policy making. When output is used as feedback, welfare is improved under cooperation in comparison with random walk taxation.

Contents

List of Figures

List of Tables

1 Introduction

Leading up to and in the wake of the implementation of the European Central Bank, much of the discussion on international policy coordination has focused on monetary policy. The potential for international fiscal policy coordination, however, has yet to draw the same amount of attention. Though a cynic may point out that fiscal authorities oft have enough difficulties coordinating policy within a country and dismiss the prospects of national fiscal authorities giving up their sovereignty, one can take comfort that the same cynic may be regretting his comments regarding the prospects of a unified European monetary authority. I shall examine whether fiscal policy coordination in a two-country setting could be beneficial. For if there are no benefits to cooperation, the issue of implementability is moot; if there are, on the other hand, possible benefits, then an earnest search for institutions to make policy coordination implementable ought to take place.

We are already seeing the beginnings of coordinated fiscal policy, though to a very limited extent. For example, the well known tariff on imported steel imposed by the US in 2002 drew immediate protests and promises for retaliatory tariffs from the EU. Tariff wars of this sort are mutually disadvantageous with the end result, under symmetrical circumstances, being simply higher prices for all and no beneficial reallocations for any. Institutions like the WTO provide extra-retaliatory recourse to unilateral fiscal policy insofar as impediments to trade are concerned. That such institutions have already come to be suggests that fiscal policy coordination, at least concerning import tariffs and free trade restrictions, is considered desirable.

But are there further opportunities for fiscal cooperation that could prove beneficial? Kim and Kim (2004) note that taxes can be used for stabilization and efficiency-impoving purposes in an open economy to over come shortcomings such as incomplete financial markets and Woodford (2001) note that fiscal policy can affect the price level even with autonomous monetary policy following a policy rule independent of fiscal policy in a cashless limiting economy. Many authors' analyses of monetary and fiscal policy are predicated on either monetary policy being optimal or both policies being coordinated. The former is unrealistic and the latter politically undesirable.

That monetary policy, given the proper instruments and information, can replicate an optimal allocation of goods in the face of certain rigidities, has been shown by many authors; recently, however, an example of the limitations of this has begun to grow in discussions of monetary policy. In a monetary union, e.g. the EMU, a central bank cannot create a monetary policy which is specific to each individual member and is forced to act aggregately, leaving open the possibility for fiscal authorities to combat country-specific shocks. It would, in my opinion, give monetary authorities too much credit to assume that they can perfectly identify the sources of distortions in the economy and analyzing the potential for fiscal policy under that assumption is misleading.

Though a benevolent dictator or social planner, in theory, may be able to achieve the best allocation of resources for all of society, he all to easily becomes a despot or a subject of discretion. It is not without good reason that many societies today have opted for a division of powers and although a social planner solution is useful as a benchmark, it is certainly limited as a policy suggestion.

I will attempt to address the issue fiscal and monetary interactions in the stabilization of open economies and the potential for cooperation amongst policy makers to improve efficiency. As fiscal policy is composed of many different elements and different authors incorporate varying amounts of these elements in their analyses of fiscal policy, what fiscal policy will consist of, needs to be defined. I shall focus on distortionary taxation and obscure from debt dynamics and seignorage transfers from central banks. Furthermore, though I include government expenditures as a demand shock, I shall not allow fiscal planners to manage these expenditures, keeping them as an exogenous process throughout.

In the vein of much literature on monetary policy and fiscal policy, though arguably to a lesser extent in the latter, policy will be represented by and analyzed through simple policy rules. The type of policy rules that I shall use will be feedback rules; that is, the choice of a particular policy, be it a tax rate or an interest rate, will be a function of macroeconomic variables. Often, policy is derived as an optimal amalgam of all elements in a model economy. Though this method derives a policy which is optimal in the true sense of the word, its resulting policies are limited by their complexity and model specificity. As has been thoroughly examined in the area of monetary policy, simple feedback rules are often capable of replicating the truly optimal solution with only minor deviations, thus giving monetary policy makers a powerful tool with which to gauge policy decisions.

Households in both countries of a two-country model will be shown to be better off when their fiscal authorities cooperate in their decisions regarding parameter values for both of the feedback rules I examine. A tax on consumption is shown to be particularly powerful through its impact on the real exchange rate, influencing consumption risk sharing despite the assumption of complete international financial markets. A unique Nash equilibrium is found for the case of feedback on technology shocks, with both policy makers raising taxes in response to positive technology shock in their respective countries. A second policy rule using output as feedback is used; I was unable to find a Nash equilibrium even when expanding the range of potential parameter values. Yet, under cooperation, this rule is able to deliver significant welfare gains over the baseline specification, lending itself to the conclusion that, under the specification used in this model, active fiscal policy contingent upon deviations in output is to be desired over passive taxation so long as authorities cooperate. Technical limitations force me to abandon policy decision interactions between both monetary and fiscal authorities. I am, however, anxious to examine these interactions and look forward to future opportunities to do so.

The remainder of this paper is organized as follows: Section 2 examines the literature contributing to an open economy New Keynesian framework, Section 3 examines the literature that expands this framework to include a more meaningful role for fiscal policy, Section 4 introduces the baseline model I use here, Section 5 solves and analyzes the baseline model, Section 6 extends the model to include endogenous tax policy setting, Section 7 discusses the results and considers improvements and further extensions, and Section 8 concludes.

2 Literature: Towards Open Economy New Keynesian Models

In recent years, much research pertaining to monetary policy and its effects on the business cycle has focused on models which draw both from Keynesian and Neoclassical sources. These models have been collectively termed "New Keynesian"(the term I shall use) or "New Neoclassical Synthesis" reflecting this fusion of previously dichotomic approaches. Nomenclature aside, these models feature some form of nominal inertia with imperfect competition (Keynesian) within the dynamic general equilibrium theory (Neoclassical). Cukierman (2005, pp. 4-7) provides a lucid account of the development in economic thought pertaining to New Keynesian models (NKM) and Yun (2004) offers one of the more frequently cited derivations of the micro-foundations of a NKM. Appealing to brevity: monetary policy in the Keynesian framework is not neutral, as nominal variables cannot adjust to neutralize actions of the monetary authority. Instead of assuming that prices are fixed, NKM's assume that there is a short-run rigidity, thus real variables will, in the short-run, have to adjust to maintain the equilibrium derived in Real Business Cycle models. Numerous techniques have been used to model this short-run rigidity: price-adjustment is costly (menu-costs), prices are set one period in advance, prices are set in a staggered fashion whereby only a portion of prices can be adjusted per period of time, and so forth. So long as some sort of nominal rigidity is implemented, cæteris paribus, monetary policy –usually by means of the nominal interest rate as the policy instrument– will matter.

That monetary policy matters begets the question of what effect it has and how it might be optimally formed. Clarida, Galí, and Gertler (1999) provide a thorough analysis of the role of monetary policy in NKM's. Central to the dynamics of NKM's is that price-setting is forward looking: i.e., in the simplest case of one-period fixed prices, price setters are not naïve but rational, they know that a price they set today will be used tomorrow as well; therefore, they will form expectations regarding future inflation and use them in determining their prices today. As agents in an economy will form rational expectations, that there may exist an optimal policy is beset with commitment problems. Kydland and Prescott (1977, pp.477-480) show that in the presence of such expectations, a policy maker attempting to decrease unemployment by increasing inflation will, in the end, only achieve the latter, with the former remaining unchanged as agents “rationally” anticipated the action of the policy maker.

To overcome this dilemma, Kydland and Prescott (1977, p. 487) suggests a policy rule, binding by authority of Congress, that, after a two year delay, dictate the reactions of fiscal and monetary authorities in the United States. Though American legislators never heeded this advice, it seems that something of the sort came nonetheless to pass. Taylor (1993, pp. 202-205) found that the actions of the Federal Reserve from 1987-1992 were, to a large extent, consistent with a simple policy rule. That this garnered interest is two-fold: a binding policy rule overcomes the dynamic inconsistency explored by Kydland and Prescott (1977) and, more importantly, this was an empirical observation. The Federal Reserve seemed to be following, if only implicitly and not exactly, a policy rule.

Despite the plethora of variations and extensions already examined, Clarida, Galí, and Gertler (1999, pp. 1701-1703) deemed six directions for future research to be “quite useful.” The second of which portends a now rapidly expanding literature: the application of New Keynesian economics to open-economy frameworks.

Though maybe not the first but arguably the most influential, Obstfeld and Rogoff (1994, p. 1) laid down the groundwork of this vein of research by ”develop[ing] a model of international policy transmission that embodies all the central elements of the intertemporal approach along with short run nominal price rigidities and explicit microfoundations of aggregate supply.” Heretofore, the literature pertaining to New Open-Economy Macroeconomics has focused mainly on monetary policy – especially its effects through nominal rigidities. As Lane (2001, p. 236) asserts, “[...] the role of nominal rigidities is most starkly illustrated in the case of monetary shocks and it is this kind of disturbance that flexible-price models are least well-equipped to handle.” In this vein of research, Obstfeld and Rogoff (1994, p. 1) laid down the groundwork by ”develop[ing] a model of international policy transmission that embodies all the central elements of the intertemporal approach along with short[-]run nominal price rigidities and explicit micro[-]foundations of aggregate supply.” Their use of one-period nominally rigid prices has been by and large supplanted in the literature by staggered pricing in accord with

Calvo (1983). More interestingly for the purposes of this paper, Obstfeld and Rogoff (1994) examine fiscal as well as monetary policy, an approach, as noted above, which has, until recently, fallen by the wayside.

In the wake of Obstfeld and Rogoff (1994), an ever-expanding literature has followed, analyzing different types of price-rigidity, alternate forms of rigidity (e.g. wage rigidity) either independently or in concert with price-rigidity, and exchange rate pass-through to mention a few. Galí and Monacelli (2002) stand arguably out from the crowd. The analysis of a small open-economy allows the authors to neglect the effects that the small open-economy has on the rest of the world (it is "too small" to affect the "rest of the world"). In comparing the specifications of three monetary policies (Consumer-Price-Index inflation targeting, domestic Producer-Price-Index inflation targeting, and an exchange-rate peg), they find a clear trade-off between the stabilization of exchange rates (real and nominal) and that of inflation and the output gap. Devereux and Engel (2000), among others, have emphasized the role of "exchange-rate pass-through," or the extent to which movements in exchange rate match those in the price of imports.

In terms of two-country open-economy models (i.e. those which do not assume "smallness"), an assumption frequently, though not necessarily innocuously, made is the limiting case where the international elasticity of substitution between goods is equal to one, e.g. (Corsetti and Pesenti 2005) and (Obstfeld and Rogoff 2000). As Benigno and Benigno (2002, p. 16) point out, "When [the elasticity of substitution between foreign and domestic goods is not equal to] 1, a closed-form solution based on log-normal shocks is no longer available and there is a need to rely on approximations of the welfare and of the structural equilibrium condition in order to characterize the optimal policy functions." This advantage to tractability notwithstanding, Pappa (2002, p. 4) analyzes the gains to monetary-policy cooperation in the absence of this assumption, stating that the value of this elasticity is "crucial for determining the incentives for policy competition." In her calibration, specific to the United States and the European Union, she reaches the conclusion that, "[a]s long as trade interdependencies between Europe and the US are as small as those experienced in the last 50 years, cooperation between the ECB and the Fed will produce little welfare gain."(Pappa 2002, p. 27)

As Tille (1999) shows, the substitutability of goods produced in different countries determines whether expansionary monetary policy will have a "beggar-thyself" or "beggar-thy-neighbor" effect, stating that, "intuitively, the [...] value of the elasticity of substitution across countries [influences] the extent of the consumption switching effect induced by the worsening of the terms of trade." (Tille 1999, p. 3) Essentially, if the relative domestic price of foreignly produced to domestically produced goods (per definition, the terms of trade) decreases, the elasticity of substitution between foreign and domestic goods determines, cæteris paribus, to what extent consumption will be shifted towards goods produced abroad.

Pappa (2002, pp. 21-23) performs a sensitivity analysis with several key parameters to determine whether any gains from cooperation are to be had. When both the elasticity of substitution across countries and the inverse of the elasticity of intertemporal substitution are equal to one, the gains from cooperation are zero. The gains from cooperation increase as the level of openness (one minus the home-bias) increases. The gains from cooperation "are negatively related to the degree of correlation between domestic and foreign shocks." (Pappa 2002, p. 23) Finally, losses from noncooperation are related negatively to the Frisch wage elasticity of labor supply.

So not only does monetary policy matter, at least in the NKM's, but in open economy versions, with appropriate specification, there can be gains from the coordination of monetary policy. A pressing macroeconomic question is then: Does this apply to fiscal policy as well?

In many NKM's, both closed and open, fiscal policy's effect is limited to subsidizing production at a constant rate either to undo the distortion originating from imperfect competition in the steady state, e.g. (Yun 2004, p. 2) and (Galí and Monacelli 2002, p. 23). Clarida, Galí, and Gertler (2002, p. 890), to "make[s] the natural level of output correspond to the efficient level in a zero inflation steady state." Despite the the cornucopia of specifications and extensions in both open and closed settings, fiscal policy in this environment has only very recently begun to be incorporated into this system of analysis.

3 Literature and Theory: Extending the NKM to Fiscal Policy

In extending the NKM specification, both open and closed economy specifications, to include fiscal policy, one certainly needs to decided on the extent of fiscal policy to be included. Most open-economy models, e.g. Beetsma and Jensen (2002), Lombardo and Sutherland (2003), Obstfeld and Rogoff (1994), Corsetti and Pesenti (2001), and Galí and Monacelli (2004), restrict their attention to government expenditures with neither distortionary taxes nor debt-dynamics being included, thereby examining the effects and stabilization possibilities of this demand shock. Canzoneri, Cumby, and Diba (2005) present a more more fully articulated model within a monetary union, studying not only government expenditures, but also the dynamics of deficits, distortionary wage and consumption taxes, and transfers to the private sector. Herz, Roeger, and Vogel (2004) examine the effects of state-dependent policy rules for government expenditure, consumption taxes, and income taxes. The majority of the models here examine fiscal policy within the confines of a monetary union, the exceptions being Lombardo and Sutherland (2003), Obstfeld and Rogoff (1994), and Corsetti and Pesenti (2001), this allows the models to abstract from the effects of exchange rates. Several thorough closed-economy NKM's that examine the role of fiscal policy have been developed, with Schmitt-Grohé and Uribe (2005) being the most comprehensive. In their model of a medium-scale macroeconomy, the authors use the Ramsey approach to determine optimal fiscal -deficits and wage, income, and profit taxes- and monetary policy in a model with multiple nominal and real rigidities. A fruitful direction for future research could be the extension of the comprehensiveness of their model into an open-economy setting.

Lombardo and Sutherland (2003, pp. 25-27), in a single-period two-country model, conclude that monetary policy can reestablish the equilibrium that would exist in the absence of rigidities regardless of fiscal policy, whereas fiscal policy is unable by itself to reestablish the same equilibrium. There are, however, gains to fiscal policy coordination if "[t]he share of steady-state government spending in output is positive" and if "[t]he supply shocks are not perfectly negatively correlated." (Lombardo and Sutherland 2003, pp. 23 and 24) It seems that government expenditures have a limited ability to stabilize the economy, though coordination of these expenditures across countries can be welfare improving.

Galí and Monacelli (2004) examine the interaction of government expenditures within a monetary union and find that "the share of government spending which is optimal from the individual countrys perspective is larger than the one perceived to be optimal from the perspective of the union as a whole." (Galí and Monacelli 2004, p. 19) Thus, fiscal authorities acting in the best interest of their respective countries will induce a sub-optimal equilibrium from the perspective of the union as a whole. This is a coordination failure intuitively similar to the one presented by Uhlig (2002), who, examining deficits instead of government expenditures, shows that in the absence of cooperation, individual fiscal authorities will want to increase deficits to improve economic conditions for their respective countries in response to a union-wide cost-push shock, more specifically the union average of a cost-push shock. Unfortunately, all fiscal authorities will pursue this measure and, in the end, "the net result is only an increase in the nominal interest rate," (Uhlig 2002, p. 16) the attempts of fiscal authorities to improve their respective countries' situation proves impotent and indeed counter-productive delivering a higher nominal interest rate than would otherwise exist. We see here that coordinating expenditures or deficits can prevent attempts of fiscal authorities from attempting to purse perceived unilaterally beneficial policies at the expense of others.

One could certainly conclude that the coordination of fiscal policy has the potential to be welfare improving, but its ability to serve as a stabilizing factor remains unclear. Kim and Kim (2004) provides a framework for distortionary taxes in an open-economy model without rigidities but with incomplete asset markets, capital accumulation and adjustments costs, and period-for-period balanced government budgets. Besides the fact that they find that tax policy should be pro-cyclical in many circumstances and that the policies obtained in a Nash equilibrium are similar to those in a cooperative one, they show that distortionary taxes can be used to move an economy towards the equilibrium that would be obtained under complete markets.

Canzoneri, Cumby, and Diba (2005) analyze fiscal policy implications in a NKM of a monetary union (specifically, the EMU). Their baseline model allows for wage-rigidities as in Erceg, Henderson, and Levin (2000) as well as capital accumulation, but they extend the specification to included distortionary taxation and debt dynamics. They divide the EMU into "[..] an

‘Average Country’, [...] a ‘High Debt Country’, and [...] a ‘Large Country’.” (Canzoneri, Cumby, and Diba 2005, pp.‘14-15) The comparison of the effects of monetary policy between “Average” and “Large” countries is rather intuitive: as the “Large” country makes up a larger proportion of the monetary union, the monetary authority will react more strongly to a country-specific technology shock in that country, yielding a larger stabilization effect. Debt dynamics turn out to be very interesting: 70%(“Average” and “Large” countries) to 80%(“High Debt” country) of the volatility in the deficit-to-GDP ratio comes from productivity shocks. The result of the Stability and Growth Pact is rather perverse in their context, “[r]ules like the [Stability and Growth Pact] try to discipline fiscal decision making by forcing fiscal policy to limit the unconditional volatility of the deficit-to-GDP ratio. But, in this context, [...] the volatility in fiscal balances that is created by productivity shocks [...] has nothing to do with a lack of fiscal discipline.” (Canzoneri, Cumby, and Diba 2005, p. 20)

As in Lombardo and Sutherland (2003) and Beetsma and Jensen (2002), I shall not tackle the problem of fiscal and monetary policy with a Ramsey approach. Though they do not explicitly explain why, two issues would be raised by a Ramsey solution: first, the Ramsey solution assumes that the policy maker can commit to his choice of an optimal path for all variables in the economy, and second, that an institution of an independent central bank was created in the first place implies that society does not find it optimal to have one institution reigning over both monetary and fiscal policy on a national level, let alone on an international one. As Uhlig (2002, p. 31) points out, “[w]hat is needed then is not the type of ‘coordination’ of current policies achievable via deals reached [among cooperating policy makers in] dark, smoke-filled rooms[, but rather] institutions that make the necessary coordination automatic in the future.” Though, in defence of Schmitt-Grohé and Uribe (2004) and Schmitt-Grohé and Uribe (2005), the authors calculate simple policy rules that minimize the distance from the Ramsey solution.

Another approach used is the maximization of a second-order accurate welfare approximation. My analysis will be similar, but I will use a first-order approximation. As I shall use the Toolkit programm of Uhlig (1999) which solves a first-order approximation, second-order terms will be ignored. I could use a second-order approximation to welfare with the results from the Toolkit, however, as Schmitt-Grohé and Uribe (2005, p. 18) point out regarding price-dispersion, "[this] would amount to leaving out certain higher-order terms while including others."

Most of the literature seems to be focused on the setting of a monetary union. And why not? Uhlig (2002, p. 32) recommends that "well-designed institutions" that "guarantee the necessary coordination between the fiscal authorities and the European Central Bank" should be a priority for policy makers in Europe. However, if fiscal policy can be welfare improving, examining institutions outside the realm of monetary unions could be fruitful. Lombardo and Sutherland (2003) show that the potential for fiscal policy in this type of model is dependent upon the actions of monetary policy makers: in the absence of cooperation among monetary authorities, cooperation over government expenditures can be welfare reducing. Thus, having two monetary authorities instead of one induces a system ripe with the potential for policy interactions.

4 The Model: Specification to Competitive Equilibrium

The economy in this model is composed of two countries, each populated by an equal number of identical households with infinite lifespans. The households in the home country (henceforth referred to as country H) are indexed $i \in [0, 1]$ and those in the foreign country (henceforth country F) $i^* \in [0, 1]$ as in Canzoneri, Cumby, and Diba (2004) and Corsetti and Pesenti (2005) among others. Other authors, e.g. Clarida, Galí, and Gertler (2002), Benigno and Benigno (2002), and Devereux and Engel (2000), emphasize the importance of differences in "mass" between the two countries, i.e. their proportion of a total population normalized to one.

4.1 Endowments and Preferences

Households

The infinitely-lived households in country H consume composite consumption C_t, work H_t hours to produce output Y_t, and hold nominal portfolios B_t. Households' utility is a function of consumption, government expenditures G_t[1], and hours worked. Household i's expected, discounted, lifetime

[1]Government expenditures need not be included directly in the household's utility

utility at time $t = 0$ is given by,

$$U(i) = E_0 \left[\sum_{t=0}^{\infty} \beta^t \, u_t(i) \right] \tag{1}$$

where

$$u_t = \frac{C_t(i)^{1-\sigma_c}}{1-\sigma_c} - \kappa_h \frac{H_t(i)^{1+\sigma_h}}{1+\sigma_h} + \kappa_g \frac{G_t^{1-\sigma_g}}{1-\sigma_g}$$

with κ_h and κ_g being relative-weight parameters for labor effort and government expenditures in utility respectively, σ_h and σ_g being the elasticity of utility with respect to labor effort and government expenditures respectively, and β being the discount factor. Furthermore, σ_c is the constant of relative risk aversion (the inverse of the elasticity of intertemporal substitution).

It is assumed that,

$$\kappa_g \geq 0 \quad \kappa_h > 0$$
$$\sigma_c, \sigma_g > 0; \; \sigma_h > 1$$
$$0 < \beta < 1$$

Note that contrary to Lombardo and Sutherland (2003, p. 10) and Canzoneri, Cumby, and Diba (2005, p. 8) , the structure has been generalized to constant relative risk aversion (CRRA) preferences similar to Beetsma and Jensen (2002, p. 9) – CRRA preferences can be deduced from their parameterization. Furthermore, real money balances have not been included in the consumers' utility functions. The importance of real money balances has been de-emphasized in many recent works, e.g. (Galí and Monacelli 2002) and (Yun 2004), as only playing the role of a unit of account. Additionally, Canzoneri, Cumby, and Diba (2005, p. 8) state, "an interest rate rule characterizes monetary policy, so there is no need to model money explicitly." Thus, monetary policy will be characterized by interest rate rules in accord therewith. See Benigno and Benigno (2002, pp. i-ii) for a detailed explanation of the cashless-limiting economy used here and in many NKM's.

The household i faces the following sequence of budget constraints,

$$\begin{aligned} (1+\tau_t^c) \int_0^1 \left[P_{H,t}(j) C_{H,t}(i)(j) + P_{F,t}(j) C_{F,t}(i)(j) \right] dj \\ + E_t \left[Q_{t,t+1} B_{t+1}(i) \right] + TR_t(i) \leq \\ (1-\tau_t^y) Y_t(i) P_{H,t}(i) + B_t(i) \end{aligned} \tag{2}$$

function; Lombardo and Sutherland (2003, p. 10) include them to "ensure [...] that welfare maximising policymakers choose a positive level of government spending. [... G]overnment consumption has no direct bearing on the use of fiscal policy as a stabilisation tool."

where $C_{H,t}(i)(j)$ and $C_{H,t}(i)(j)$ are household i's consumption of individual, differentiated domestic and foreign goods, respectively, $P_{H,t}(j)$ and $P_{F,t}(j)$ the respective prices of the same, $Q_{t,t+1}$ [2] the stochastic discount factor for nominal payoffs from period t to $t+1$, $B_t(i)$ the nominal portfolio held, and $TR_t(i)$ the nominal lump-sum tax paid by household i. Furthermore, τ_t^c and τ_t^y are the tax rates on consumption and production, respectively.

Upon imposing the transversality condition

$$\lim_{s\to\infty} E_0\left[\sum_{t=0}^{\infty} Q_{0,t} B_t\right] = 0$$

to rule out Ponzi-schemes and such that the expected sum of lifetime expenditures does not exceed that of lifetime income,

$$E_0\left[\sum_{t=0}^{\infty} Q_{0,t}\left((1+\tau_t^c)\int_0^1 \left[P_{H,t}(j)C_{H,t}(i)(j) + P_{F,t}(j)C_{F,t}(i)(j)\right] dj \right.\right.$$
$$\left.\left. + TR_t(i)\right)\right] = E_0\left[\sum_{t=0}^{\infty} Q_{0,t}\left((1-\tau_t^y)Y_t(i)P_{H,t}(i)\right)\right]$$

Households in country F face analogous constraints which will not be explicitly stated.

Fiscal Authorities

A fiscal authority in country H levies taxes τ_t^y on production and τ_t^c on consumption in country H and imposes lump-sum taxes TR_t on the households in country H. It consumes individual differentiated goods $G_t(j)$ produced in country H, paying the price $P_{H\,t}j$ for each good j. Finally, it issues nominal bonds b_t. It faces the following sequence of budget constraints,

$$\int_0^1 \left[P_{H,t}(j)G_{H,t}(j)\right] dj + R_{t-1}\int_0^1 b_{t-1}(i)di \leq$$
$$\tau_t^c \int_0^1 \left[\int_0^1 \left[P_{H,t}(j)C_{H,t}(i)(j) + P_{F,t}(j)C_{F,t}(i)(j)\right] dj\right] di$$
$$+\tau_t^y \int_0^1 \left[Y_t(i)P_{H,t}(i)\right] di + \int_0^1 b_t(i)di + \int_0^1 TR_t(i)di$$

where R_t is the gross nominal interest rate, assumed to be > 0 at all times.

[2] This is analogous to the portfolio and payoff scheme used by Galí and Monacelli (2002, p. 3) and Galí and Monacelli (2004, p. 4). In contrast, Pappa (2002, p. 6) assumes state-contingent nominal bonds. Beetsma and Jensen (2002) and Lombardo and Sutherland (2003) forgo presenting an analogy to the foregoing and assume the results of Benigno and Benigno (2004, pp. 298-300) concerning portfolio decisions.

It will be assumed that fiscal authorities balance their budgets in every period[3]; this amounts to denying fiscal authorities the use of deficit finance. The foregoing can thus be rewritten as,

$$\begin{aligned} \int_0^1 \left[P_{H,t}(j)G_{H,t}(j)\right] dj = \\ \tau_t^c \int_0^1 \left[\int_0^1 \left[P_{H,t}(j)C_{H,t}(i)(j) + P_{F,t}(j)C_{F,t}(i)(j)\right] dj\right] di \\ +\tau_t^y \int_0^1 \left[Y_t(i)P_{H,t}(i)\right] di + \int_0^1 TR_t(i)di \end{aligned} \tag{3}$$

Essentially, the presence of endogenous lump-sum taxation makes deficit financing irrelevant. Assuming that nominal bonds issued by the government are amongst the assets held by the households, the households are ambivalent vis-à-vis the financing (or disbursement) of fiscal shortfalls (windfalls) by debt-issuance or lump-sum taxation: the Ricardian equivalence result, see e.g. Romer (2001, pp. 535-537). Notice that with shortfalls (windfalls) the net-of-distortionary-tax government deficit (surplus) is meant, Ljungqvist and Sargent (2004, p.325) note that this ia a case where "the government [has] access to too many kinds of taxes, [since] lump-sum taxes [are] available, the government typically should not use [...] the [distortionary] taxes." But as they go on to note, this framework allows one to "analyze how the various taxes distort production and consumption decisions," exactly what this model will try to accomplish.

4.2 Consumption Allocation

Consumers in country H assemble composite consumption using the CES aggregator,

$$C_t \equiv \left[(1-\theta)^{\frac{1}{\omega}} C_{H,t}{}^{\frac{\omega-1}{\omega}} + \theta^{\frac{1}{\omega}} C_{F,t}{}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{4}$$

where $1-\theta$ allows for home-bias in consumption, with $0 \leq \theta \leq 1$, and $\omega \geq 1$[4]is the elasticity of international goods substitution. "For $[\theta < \frac{1}{2}]$,

[3]Though this omits a certainly important and interesting dynamic that could otherwise be included in the model, I will leave the exploration of government deficits to further investigations; e.g. Kim and Kim (2004) follow this route as well.

[4]Much of the literature, e.g. Beetsma and Jensen (2002), Lombardo and Sutherland (2003), and Benigno and Benigno (2004), assumes $\omega = 1$, unitary elasticity of substitution, which provides more tractable results, but as Pappa (2002, p. 4) points out, "when this elasticity is different from one, terms of trade movements affect relative consumption movements and national policymakers have incentives to use strategically the terms of trade to improve domestic relative welfare."

domestic consumers will always demand relatively more domestic goods than foreign consumers." (Pappa 2002, p. 6)

The consumption bundles bound for country H are defined using Dixit-Stiglitz-type constant-elasticity-of-substitution (CES) aggregators (Dixit and Stiglitz 1977),

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{5}$$

$$C_{F,t} \equiv \left(\int_0^1 C_{F,t}(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{6}$$

where $\eta > 1$ measures the elasticity of substitution between differentiated goods within a given country. In accord with Pappa (2002, p. 6), less substitutability between than within countries will be assumed, i.e. $\omega \leq \eta$.

Following Beetsma and Jensen (2002, p. 10), the allocation of consumption to differentiated goods across countries will be achieved through the following three steps:

1. Household i determines $C_t(i)$

2. Given $C_t(i)$, household i divides its consumption between bundles of domestic and foreign consumption goods $C_{H,t}$ and $C_{F,t}$ by means of cost minimization, determining its individual consumption of the same, $C_{H,t}(i)$ and $C_{F,t}(i)$

3. Given $C_{H,t}(i)$ ($C_{F,t}(i)$), household i divides its consumption of $C_{H,t}(i)$ ($C_{F,t}(i)$) amongst differentiated domestic (foreign) goods $C_{H,t}(j), j \in [0,1]$ ($C_{F,t}(j), j \in [0,1]$) by means of cost minimization, determining its individual consumption of differentiated domestic (foreign) goods, $C_{H,t}(i)(j)$ ($C_{F,t}(i)(j)$)

Solving backwards, household i minimizes its costs associated with purchase of differentiated, domestic goods $C_{H,t}(j)$ to assemble a given amount of the domestic consumption bundle $C_{H,t}$. All households in country H have identical preferences regarding differentiated goods; that is, they are all cost minimizers endowed with the same assembly technology (5). As such, they will all face the same price for the domestic consumption bundle $C_{H,t}$ and purchase differentiated, domestic goods according to the same demand function. These are given by [5],

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\eta} dj\right)^{\frac{1}{1-\eta}} \tag{7}$$

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\eta} C_{H,t} \tag{8}$$

with the analogous equations for the domestic bundle of foreign goods being,

$$P_{F,t} = \left(\int_0^1 P_{F,t}(j)^{1-\eta} dj\right)^{\frac{1}{1-\eta}} \tag{9}$$

$$C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\eta} C_{F,t} \tag{10}$$

Now taking composite consumption C_t to be given (it will be derived later through inter-temporal maximization), households assemble composite consumption using the assembly technology (4). One should notice that consumers will face the same price and have the same preferences following the logic used in deriving (7-10). The price of composite consumption and demand for the domestic and foreign consumption bundles in country H are[6],

$$P_t = \left[(1-\theta)\, {P_{H,t}}^{1-\omega} + \theta {P_{F,t}}^{1-\omega}\right]^{\frac{1}{1-\omega}} \tag{11}$$

$$C_{H,t} = (1-\theta)\left(\frac{P_{H,t}}{P_t}\right)^{-\omega} C_t \tag{12}$$

$$C_{F,t} = \theta\left(\frac{P_{F,t}}{P_t}\right)^{-\omega} C_t \tag{13}$$

Analogous expressions hold in country F[7], where,

$$C_t^* \equiv \left[(1-\theta)^{\frac{1}{\omega}} {C^*_{F,t}}^{\frac{\omega-1}{\omega}} + \theta^{\frac{1}{\omega}} {C^*_{H,t}}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{14}$$

$$C^*_{F,t} \equiv \left(\int_0^1 C^*_{F,t}(j)^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}}; \quad C^*_{H,t} \equiv \left(\int_0^1 C^*_{H,t}(j)^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}} \tag{15}$$

[5] Please see Appendix A.1 for derivation.

[6] In Appendix A.1, the minimization problem is introduced, the solution method is analogous to that of (7-8).

[7] A small remark concerning notation is in order here: a superscripted asterisk denotes a variable in country F, whereas the subscripted "F" denotes a variable originating in country F. E.g., $C^*_{F,t}$ is the bundle of goods produced in Country F bound for consumption in Country F. Though as such, it is "domestic" from the perspective of Country F, it is the foreign consumption bundle bound for consumption in the foreign country according to my notation. To put it another way, the terms "home" and "foreign" refer to the countries H and F respectively, and *not* to the terms "home" (likewise "domestic") and "foreign" from the perspective of the respective countries.

$$P_t^* = \left[(1-\theta) \, P_{F,t}^{*\,1-\omega} + \theta P_{H,t}^{*\,1-\omega} \right]^{\frac{1}{1-\omega}} \tag{16}$$

$$P_{F,t}^* = \left(\int_0^1 P_{F,t}^*(j)^{1-\eta} dj \right)^{\frac{1}{1-\eta}}; \quad P_{H,t}^* = \left(\int_0^1 P_{H,t}^*(j)^{1-\eta} dj \right)^{\frac{1}{1-\eta}} \tag{17}$$

$$C_{F,t}^* = (1-\theta) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\omega} C_t^* \, ; \quad C_{H,t}^* = \theta \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\omega} C_t^* \tag{18}$$

$$C_{F,t}^*(j) = \left(\frac{P_{F,t}^*(j)}{P_{F,t}^*} \right)^{-\eta} C_{F,t}^* \, ; \quad C_{H,t}^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\eta} C_{H,t}^* \tag{19}$$

In both countries, governments are assumed to purchase only those goods produced in their respective countries. Furthermore, they use the following aggregators to assemble their composite expenditures,

$$G_t \equiv \left(\int_0^1 G_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}$$

and

$$G_t^* \equiv \left(\int_0^1 G_t^*(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}$$

where $G_t(j)$ and $G_t^*(j)$ are individual goods produced in country H and country F, respectively.

Cost minimization, following the method in Appendix 1 in Section A.1, yields the prices (7) and (17). Government demand for individual goods is,

$$G_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\eta} G_t \tag{20}$$

in the home country, and,

$$G_t^*(j) = \left(\frac{P_{F,t}^*(j)}{P_{F,t}^*} \right)^{-\eta} G_t^* \tag{21}$$

in the foreign country. Total government expenditures will be considered exogenous and given by,

$$ln(G_t) = (1-\rho_G) ln(\bar{G}) + \rho_G ln(G_{t-1}) + \epsilon_t^g \tag{22}$$

$$ln(G_t^*) = (1-\rho_{G^*}) ln(\bar{G^*}) + \rho_{G^*} ln(G_{t-1}^*) + \epsilon_t^{g^*} \tag{23}$$

where,

$$\epsilon_t^g \sim i.i.d \mathcal{N}(0, \sigma_\epsilon^g), \;\; \epsilon_t^{g*} \sim i.i.d \mathcal{N}(0, \sigma_\epsilon^{g*}) \tag{24}$$

Combining (20), (8), and the second equation in (19), total demand for an individual good produced in country H can be represented by:

$$Y_t^d(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\eta} [G_t + C_{H,t}] + \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\eta} C_{H,t}^* \tag{25}$$

It is assumed that

> [t]here are no impediments or costs to trade between the countries. Let $[e_t]$ be the nominal exchange rate, defined as the home-currency price of foreign currency, $[P_{H,t}(j)]$ the domestic-currency price of good [j], and $[P^*_{H,t}(j)]$ the price of the same good in foreign currency. Then the law of one price holds for every good. (Obstfeld and Rogoff 1994, p.3)

Notice that since this model is a "cashless limiting" model, "the hypothetical limiting case of an economy in which financial innovation has proceeded to the extent that available seignorage revenues are negligible," and currency need not be modelled explicitly. (Woodford 2001, p. 5) Thus,

$$P_{H,t}(j) = e_t P^*_{H,t}(j) \tag{26}$$

$$P_{F,t}(j) = e_t P^*_{F,t}(j) \tag{27}$$

Combining (26) with (7) and the second equation in (17) yields,

$$P_{H,t} = \left(\int_0^1 \left[e_t P^*_{H,t}(j) \right]^{1-\eta} dj \right)^{\frac{1}{1-\eta}} = e_t \left(\int_0^1 P^*_{H,t}(j)^{1-\eta} dj \right)^{\frac{1}{1-\eta}} = e_t P^*_{H,t} \tag{28}$$

Likewise, combining (27) with (9) and the first equation in (17) delivers,

$$P_{F,t} = e_t P^*_{F,t} \tag{29}$$

Notice that $P_t \neq e_t P^*_t$, from (29), (28), and (16),

$$\begin{aligned} P^*_t &= \frac{1}{e_t} \left[(1-\theta) P_{F,t}{}^{1-\omega} + \theta P_{H,t}{}^{1-\omega} \right]^{\frac{1}{1-\omega}} \\ &\neq \frac{1}{e_t} \left[(1-\theta) P_{H,t}{}^{1-\omega} + \theta P_{F,t}{}^{1-\omega} \right]^{\frac{1}{1-\omega}} \end{aligned}$$

so long as $\theta \neq \frac{1}{2}$ or $\omega \neq 1$. Thus, as Pappa (2002, p. 7) notes, purchasing power will not hold.

The ratio of prices for foreign goods and domestic goods in country H, terms of trade, is defined as,

$$s_t = \frac{P_{F,t}}{P_{H,t}} \tag{30}$$

Using (26) and (28), (25) can be rewritten as,

$$Y^d_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\eta} \left[G_t + C_{H,t} + C^*_{H,t} \right] \tag{31}$$

Analogously, total demand for an individual good produced in country F can be represented by,

$$Y_t^{*d}(j) = \left(\frac{P_{F,t}^*(j)}{P_{F,t}^*}\right)^{-\eta} \left[G_t^* + C_{F,t}^* + C_{F,t}\right] \tag{32}$$

This assumption implies perfect exchange-rate pass-through. In this case, producer-currency pricing is assumed; that is, producers set their prices in the currency of their country and the prices of imports fluctuate one-to-one with fluctuations in the exchange rate. Among others, Corsetti and Pesenti (2005) and Devereux and Engel (2002) have brought attention to the fact that perfect exchange-rate pass-through is not in line with empirical findings. Corsetti and Pesenti (2005) also emphasizes that its converse (the complete absence of exchange-rate pass-through found in local-currency pricing) is an extreme assumption and studies intermediate levels of pass-through. That notwithstanding, producer-currency pricing will be assumed in this model.

4.3 Households' Maximization Problem

After taking the optimality conditions of consumption allocation yielded through cost minimization and the definitions of the consumption bundles (all of which were presented in Section 4.2) into account, the sequence of budget restrictions facing household i in country H can be rewritten as follows:

$$(1+\tau_t^c)P_tC_t(i)+E_t\left[Q_{t,t+1}B_{t+1}(i)\right]+TR_t(i) = (1-\tau_t^y)Y_t(i)P_{H,t}(i)+B_t(i) \tag{33}$$

The problem facing household i is to maximize its utility (1) subject to its budget constraint (33). This maximization problem can be represented by the following Lagrangian,

$$\begin{aligned}\mathcal{L} &= E_0\sum_{t=0}^{\infty}\beta^t\Bigg[\frac{C_t(i)^{1-\sigma_c}}{1-\sigma_c} - \frac{\kappa_h H_t(i)^{1+\sigma_h}}{1+\sigma_h} + \frac{\kappa_g G_t^{1-\sigma_g}}{1-\sigma_g} \\ &+ \lambda_t\left((1-\tau_t^y)Y_t(i)P_{H,t}(i) + B_t(i) + TR_t(i)\right. \\ &- \left.(1+\tau_t^c)P_tC_t(i) - E_t\left[Q_{t,t+1}B_{t+1}(i)\right]\right)\Bigg]\end{aligned} \tag{34}$$

The first-order conditions include,

$$\frac{\partial \mathcal{L}}{\partial C_t(i)} \stackrel{!}{=} 0 = C_t(i)^{-\sigma_c} - \lambda_t(i)(1+\tau_t^c)P_t \Rightarrow \lambda_t(i) = \frac{C_t(i)^{-\sigma_c}}{(1+\tau_t^c)P_t} \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} \stackrel{!}{=} 0 = -\lambda_t(i) Q_{t,t+1} + \beta \lambda_{t+1}(i) \tag{36}$$

Following Corsetti and Pesenti (2002, p. 6), the subscript i will be dropped by noting that all households in country H have identical first-order conditions and interpreting variables as per capita. Thus,

$$\lambda_t = \frac{C_t^{-\sigma_c}}{(1+\tau_t^c) P_t} \tag{37}$$

$$0 = -\lambda_t Q_{t,t+1} + \beta \lambda_{t+1} \tag{38}$$

Substituting (37) into (38) to eliminate λ_t, and rearranging yields the stochastic discount factor,

$$Q_{t,t+1} = \beta \frac{C_{t+1}^{-\sigma_c}}{C_t^{-\sigma_c}} \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} \frac{1}{\pi_{t+1}} \tag{39}$$

where,

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{40}$$

has been defined as gross consumer price index (CPI) inflation.

Country F's analogs to equations (37), (39), and (40) are, respectively:

$$0 = C_t^{*-\sigma_c} - \lambda_t^* (1+\tau_t^{c*}) P_t^* \tag{41}$$

$$Q_{t,t+1}^* = \beta \frac{C_{t+1}^{*\ -\sigma_c}}{C_t^{*-\sigma_c}} \frac{(1+\tau_t^{c*})}{(1+\tau_{t+1}^{c*})} \frac{1}{\pi_{t+1}^*} \tag{42}$$

$$\pi_t^* = \frac{P_t^*}{P_{t-1}^*} \tag{43}$$

4.4 Intra- and International Financial Markets

Following Galí and Monacelli (2002, p. 4), taking conditional expectations on both sides of (39) yields,

$$1 = \beta R_t E_t \left[\frac{C_{t+1}^{-\sigma_c}}{C_t^{-\sigma_c}} \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} \frac{1}{\pi_{t+1}} \right] \tag{44}$$

and likewise for (42),

$$1 = \beta R_t^* E_t \left[\frac{C_{t+1}^{*\ -\sigma_c}}{C_t^{*-\sigma_c}} \frac{(1+\tau_t^{c*})}{(1+\tau_{t+1}^{c*})} \frac{1}{\pi_{t+1}^*} \right] \tag{45}$$

In essence, if intranational financial markets are complete, then one can rule out arbitrage possibilities. Thusly, the gross, risk-free nominal interest rate (equivalently, the gross nominal yield on a one-period discount bond (Clarida, Galí, and Gertler 2002, p. 883)) must be equal to the inverse of the stochastic discount factor[8]

As it is assumed that international financial markets are complete, households in country F must be allowed to hold domestic portfolios and households in country H foreign ones. The consequence of this market completeness is international risk sharing. The inclusion of foreign (home) portfolios denominated in foreign (home) currency in the home (foreign) country's budget constraint yields two identical conditions, the first of which will be derived here with the second being left to the reader.

The Lagrangian for the foreign country's representative consumer pertinent to bond-holdings is given by:

$$\mathcal{L}_{*\,B,B^*} = E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^* \Big(B_t^* - Q_{t,t+1}^* B_{t+1}^* + \frac{1}{e_t} \left[B_t - Q_{t,t+1} B_{t+1} \right] \Big)$$

where e_t is the nominal exchange rate. The first-order condition pertaining to domestic bonds is

$$\frac{\partial \mathcal{L}_{*\,B,B^*}}{\partial B_{t+1}} \stackrel{!}{=} 0 = -\lambda_t^* \frac{Q_{t,t+1}}{e_t} + \beta \frac{\lambda_{t+1}^*}{e_{t+1}} \tag{46}$$

Combining this equation with (38) yields,

$$\beta\, \frac{\lambda_{t+1}}{\lambda_t} \;=\; \beta\, \frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{e_t}{e_{t+1}}$$

Substituting (37) and (41) for λ and λ^* in the foregoing yields,

$$\beta\, \frac{C_{t+1}^{-\sigma_c}}{C_t^{-\sigma_c}} \frac{(1+\tau_t^c)P_t}{(1+\tau_{t+1}^c)P_{t+1}} \;=\; \beta\, \frac{{C_{t+1}^*}^{-\sigma_c}}{{C_t^*}^{-\sigma_c}} \frac{(1+\tau_t^{c*})P_t^*}{(1+\tau_{t+1}^{c*})P_{t+1}^*} \frac{e_t}{e_{t+1}} \tag{47}$$

Rearranging, taking conditional expectations, recalling that $\pi_t = \frac{P_t}{P_{t-1}}$ and $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$, and using (44) and (45) delivers the following version of the uncovered interest parity condition[9],

$$E_t \left[Q_{t,t+1} \left(R_t - R_t^* \frac{e_{t+1}}{e_t} \right) \right] = 0 \tag{48}$$

Defining the real exchange rate as

$$q_t = e_t \frac{P_t^*}{P_t} \tag{49}$$

[8] See (Yun 2004, p. 4) and (Schmitt-Grohé and Uribe 2005, p. 10)

[9] Galí and Monacelli (2002, p. 7) provides an identical expression: merely the underlying relationships, i.e. the definition of the stochastic discount factor, defining the variables differ.

and solving (47) recursively delivers[10],

$$q_t = \Xi \frac{C_t^{*-\sigma_c}}{C_t^{-\sigma_c}} \frac{(1+\tau_t^c)}{(1+\tau_t^{c^*})} \tag{50}$$

where Ξ is a constant dependent upon initial conditions, equaling,

$$\Xi = e_0 \frac{P_0^*}{P_0} \frac{C_0^{*-\sigma_c}}{C_0^{-\sigma_c}} \frac{(1+\tau_0^c)}{(1+\tau_0^{c^*})} \tag{51}$$

4.5 Production

Individual Households are assumed to be monopolistic producers of individual differentiated goods[11]. Household i has access to the following production function,

$$Y_t(i) = Z_t H_t^{d\,\gamma}(j) \tag{52}$$

where $Y_t(i)$ denotes the production of differentiated good i, $H_t^d(i)$ the factor input labor services demanded (and supplied) by household i, and Z_t a stochastic, exogenous productivity shock common to the domestic market. An appropriate assumption regarding γ would assert that labor is not an impediment to production, i.e. $\gamma \geq 0$. Furthermore, assuming decreasing or constant returns to scale, $\gamma \leq 1$. Z_t is assumed to follow the law of motion given by,

$$ln\, Z_t = \rho_z ln\, Z_{t-1} + \epsilon_t^z \tag{53}$$

with its foreign analog being equivalently defined as,

$$ln\, Z_t^* = \rho_{z*} ln\, Z_{t-1}^* + \epsilon_t^{z*} \tag{54}$$

where,

$$\epsilon_t^z \sim i.i.d\mathcal{N}(0, \sigma_\epsilon^z), \quad \epsilon_t^{z*} \sim i.i.d\mathcal{N}(0, \sigma_\epsilon^{z*})$$

Household i faces the following demand constraint, introduced earlier as (31):

$$Y_t^d(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\eta} Y_t^d \tag{55}$$

where $Y_t^d = C_{H,t} + C_{H,t}^* + G_t$.

[10] This can be found, e.g., in Galí and Monacelli (2002, p. 6).

[11] This is the arrangement used by Obstfeld and Rogoff (1994, p. 3), whose agents are described by Lane (2001, p. 237) as "yeoman-farmers".

4.6 Price Setting

Staggered price setting in the fashion of Calvo (1983) is assumed. Thus, a fraction , say $1-\alpha$, of producers are allowed to set their prices optimally in period t, the remaining fraction, α, maintain their prices from the previous period. The probability that a producer will be allowed to set its price anew is independent of when in the past it last adjusted its price. Thus, every producers that sets its price in period t will still be using that price in period $t+s$ with the probability α^s. Note that in this model, household i is the monopolistic producer of good i. Accordingly, a household setting its price $\tilde{P}_{H,t}(i)$ in period t will do so by maximizing its expected present discounted utility from production according to a version of (34) modified to reflect this probability,

$$\mathcal{L}_{H(Y_t(i)),Y_t(i)} = E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left[\lambda_{t+s}\left((1-\tau^y_{t+s})Y_{t+s}(i)\tilde{P}_{H,t}(i)\right) - \frac{\kappa_h H_{t+s}(i)^{1+\sigma_h}}{1+\sigma_h}\right]$$

Assuming producers meet demand for goods, $Y_t(i) = Y^d_t(i)$ where $Y^d_t(i)$ is given by (55), and having noted that in order to so, producers must choose $H_t(i)$ such that (52) holds, the foregoing can be rewritten as,

$$\mathcal{L}_{Y_t(i)} = E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left[\lambda_{t+s}(1-\tau^y_{t+s})\left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t+s}}\right)^{-\eta} Y^d_{t+s}\tilde{P}_{H,t}(i) \right.$$
$$\left. - \frac{\kappa_h}{1+\sigma_h}\left(\frac{\left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t+s}}\right)^{-\eta} Y^d_{t+s}}{Z_{t+s}}\right)^{\frac{1+\sigma_h}{\gamma}}\right]$$

The preceding equation can be rewritten as,

$$\mathcal{L}_{Y_t(i)} = E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left[\lambda_{t+s}(1-\tau^y_{t+s})\Gamma_{t+s}\tilde{P}_{H,t}(i)^{1-\eta}\right.$$
$$\left. - \frac{\kappa_h}{1+\sigma_h}\tilde{P}_{H,t}(i)^{\frac{-\eta(1+\sigma_h)}{\gamma}}\left(\frac{\Gamma_{t+s}}{Z_{t+s}}\right)^{\frac{1+\sigma_h}{\gamma}}\right]$$

where,

$$\Gamma_t = P_{H,t}{}^{\eta} Y^d_t$$

Maximizing the household's utility from production by maximizing $\tilde{P}_{H,t}(i)$ in the foregoing delivers the following first-order condition,

$$\frac{\partial \mathcal{L}_{Y_t(i)}}{\partial \tilde{P}_{H,t}(i)} \overset{!}{=} 0 = E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left[(1-\eta)\,\lambda_{t+s}(1-\tau^y_{t+s})\Gamma_{t+s}\tilde{P}_{H,t}(i)^{-\eta}\right.$$

$$+\frac{\eta\left(1+\sigma_h\right)}{\gamma}\frac{\kappa_h}{1+\sigma_h}\tilde{P}_{H,t}(i)^{\frac{-\eta(1+\sigma_h)}{\gamma}-1}\left(\frac{\Gamma_{t+s}}{Z_{t+s}}\right)^{\frac{1+\sigma_h}{\gamma}}\Bigg] \tag{56}$$

which can be rewritten as,

$$\tilde{P}_{H,t}(i)=\frac{\eta}{\eta-1}\frac{E_t\sum_{s=0}^{\infty}\left(\alpha\beta\right)^s\left[\frac{\kappa_h}{\gamma}Y_{t+s}^d(i)^{\frac{1+\sigma_h}{\gamma}}Z_{t+s}^{-\frac{1+\sigma_h}{\gamma}}\right]}{E_t\sum_{s=0}^{\infty}\left(\alpha\beta\right)^s\left[\lambda_{t+s}(1-\tau_{t+s}^y)Y_{t+s}^d(i)\right]} \tag{57}$$

This is identical with the condition derived by Benigno (2001, p. 9) and Beetsma and Jensen (2002, p. 12) with the exception that in this model, their production subsidy, here tax, is not constant. It is noteworthy that Pappa (2002, p. 11) derives a different condition despite the fact that that paper features yeoman-farmers (producer-consumers) like the two just mentioned. The condition here (as well as that of the first two authors mentioned) has the producer maximizing utility from production through price setting restricted by λ_{t+s}, the Lagrange multiplier or, from (37), the value of net-of-taxes real marginal consumption. Pappa (2002, p. 7), in a state-coningent framework, has the producer-consumers acting as firms, maximizing nominal revenues discounted by the stochastic discount factor: that the producer-consumers shall consume plays a role in the price setting mechanism only insofar as the stochastic discount factor is concerned.

Examining (56), one notices that the only producer-specific variable that enters into the price-setting condition is the producer-specific price itself; thus, all producers, when given the chance, will set the same price. Furthermore, following Schmitt-Grohé and Uribe (2005, p. 14), (57) will be solved recursively. Besides making this equation more palatable, this will eliminate the summations in the same. To that end, define,

$$\tilde{P}_{H,t}=\frac{\eta}{\eta-1}\frac{x_{1,t}}{x_{2,t}} \tag{58}$$

where,

$$\begin{aligned} x_{1,t} &= E_t\sum_{s=0}^{\infty}\left(\alpha\beta\right)^s\left[\frac{\kappa_h}{\gamma}Y_{t+s}^{d\ \frac{1+\sigma_h}{\gamma}}Z_{t+s}^{-\frac{1+\sigma_h}{\gamma}}\right] \\ &= \frac{\kappa_h}{\gamma}Y_t^{d\ \frac{1+\sigma_h}{\gamma}}Z_t^{-\frac{1+\sigma_h}{\gamma}}+\alpha\beta E_t\left[x_{1,t+1}\right] \end{aligned} \tag{59}$$

and,

$$\begin{aligned} x_{2,t} &= E_t\sum_{s=0}^{\infty}\left(\alpha\beta\right)^s\left[\lambda_{t+s}(1-\tau_{t+s}^y)Y_{t+s}^d\right] \\ &= \lambda_t(1-\tau_t^y)Y_t^d+\alpha\beta E_t\left[x_{2,t+1}\right] \end{aligned}$$

Using (37), λ_t can be eliminated from the preceding, delivering,

$$x_{2,t} = \frac{(1-\tau_t^y)}{(1+\tau_t^c)} \frac{C_t^{-\sigma_c} Y_t^d}{P_t} + \alpha\beta E_t\left[x_{2,t+1}\right] \tag{60}$$

As this price is common to all producers in country H, all prices of individual domestic goods in this system of staggered price-setting can take only one of two values in period t ($\tilde{P}_{H,t}$ or the price it took the period theretofore), the price of the domestic goods bundle $P_{H,t}$ can be thusly expressed as:

$$P_{H,t} = \left((1-\alpha)\tilde{P}_{H,t}^{1-\eta} + \alpha P_{H,t-1}^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{61}$$

The foreign analogs to (58), (59), (60), and (61) are,

$$\tilde{P^*}_{F,t} = \frac{\eta}{\eta-1} \frac{x_{1,t}^*}{x_{2,t}^*} \tag{62}$$

$$x_{1,t}^* = \frac{\kappa_h}{\gamma} {Y^*}_t^{d \frac{1+\sigma_h}{\gamma}} {Z^*}_t^{-\frac{1+\sigma_h}{\gamma}} + \alpha\beta E_t\left[x_{1,t+1}^*\right] \tag{63}$$

$$x_{2,t}^* = \frac{(1-\tau_t^{y*})}{(1+\tau_t^{c*})} \frac{{C^*}_t^{-\sigma_c} {Y^*}_t^d}{P_t^*} + \alpha\beta E_t\left[x_{2,t+1}^*\right] \tag{64}$$

$$P_{F,t}^* = \left((1-\alpha)\tilde{P^*}_{F,t}^{1-\eta} + \alpha {P^*}_{F,t-1}^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{65}$$

4.7 Fiscal and Monetary Authorities

Fiscal Authorities

It is assumed that fiscal authorities balance their budgets every period. Using the conditions derived in Section 4.2 and interpreting variables as per capita ($\int_0^1 TR_t(i)di = TR_t,\ \int_0^1 C_t(i)di = C_t$), allows the government budget constraint (3) to be rewritten as,

$$P_{H,t}G_t = \tau_t^c P_t C_t + \tau_t^y Y_t P_{H,t} + TR_t \tag{66}$$

where it has been noted that,

$$\int_0^1 \left[Y_t(i)P_{H,t}(i)\right] di = \int_0^1 \left[Y_t \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\eta} P_{H,t}(i)\right] di = Y_t P_{H,t}$$

by virtue of the fact that (7) can be rearranged to show,

$$P_{H,t}^{1-\eta} = \int_0^1 P_{H,t}(j)^{1-\eta} dj \Rightarrow P_{H,t} = \frac{\int_0^1 P_{H,t}(j)^{1-\eta} dj}{P_{H,t}^{-\eta}}$$

In the baseline model, taxes will be assumed exogenously given by the following equations,

$$ln(\tau_t^c) = (1 - \rho_{\tau^c})ln(\bar{\tau^c}) + \rho_{\tau^c} ln(\tau_{t-1}^c) + \epsilon_t^{\tau^c} \tag{67}$$

$$ln(\tau_t^y) = (1 - \rho_{\tau^y})ln(\bar{\tau^y}) + \rho_{\tau^y} ln(\tau_{t-1}^y) + \epsilon_t^{\tau^y} \tag{68}$$

where

$$\epsilon_t^{\tau^c} \sim i.i.d\mathcal{N}(0, \sigma_\epsilon^{\tau^c}), \;\; \epsilon_t^{\tau^y} \sim i.i.d\mathcal{N}(0, \sigma_\epsilon^{\tau^y})$$

The analogous expressions for country F are

$$P_{F,t}^* G_t^* = \tau_t^{c*} P_t^* C_t^* + \tau_t^{y*} Y_t^* P_{F,t}^* + TR_t^* \tag{69}$$

$$ln(\tau_t^{c*}) = (1 - \rho_{\tau^{c*}})ln(\bar{\tau^{c*}}) + \rho_{\tau^{c*}} ln(\tau_{t-1}^{c*}) + \epsilon_t^{\tau^{c*}} \tag{70}$$

$$ln(\tau_t^{y*}) = (1 - \rho_{\tau^{y*}})ln(\bar{\tau^{y*}}) + \rho_{\tau^{y*}} ln(\tau_{t-1}^{y*}) + \epsilon_t^{\tau^{y*}} \tag{71}$$

where

$$\epsilon_t^{\tau^{c*}} \sim i.i.d\mathcal{N}(0, \sigma_\epsilon^{\tau^{c*}}), \;\; \epsilon_t^{\tau^{y*}} \sim i.i.d\mathcal{N}(0, \sigma_\epsilon^{\tau^{y*}})$$

Monetary Authorities

Monetary policies will be a standard Taylor Rule as proposed by Taylor (1993, p. 202), given here in log-linear form for this model as:

$$\bar{R}\hat{R}_t = \phi_y \hat{y}_t + \phi_\pi \hat{\pi}_t \tag{72}$$

for country H and,

$$\bar{R}^* \hat{R}_t^* = \phi_y^* \hat{y}_t^* + \phi_\pi^* \hat{\pi}_t^* \tag{73}$$

for country F, where following Taylor (1993, p. 202),

$$\phi_y = \phi_y^* = 0.5$$
$$\phi_\pi = \phi_\pi^* = 1.5$$

This is described by Clarida, Galí, and Gertler (1999, p.1672) as a "lean against the wind" policy, that is, the monetary authority should raise the interest rate by more than one in response to a deviation of inflation from its target level (here steady-state level, shown later to be equal to one), thus 'counter-balancing' inflation. Notice that this specification of monetary policy is in some sense sub-optimal in an open economy setting: Ball (1998,

p. 6) points out that monetary policy in open economies amounts to setting a combination of the nominal interest rate and the real exchange rate; Clarida, Galí, and Gertler (1998, pp. 1031-1038, 1045, and 1049) find empirically that the monetary policy of the German Bundesbank and the American Federal Reserve Bank, among others, can be represented by an interest rate for each reacting to inflation, output, and the real exchange rate (in the case of the Bundesbank).

4.8 World Resource Constraint

Aggregating (33) and its foreign analog, requiring that nominal portfolios are in net zero international supply ($\int_0^1 B_t(i)di + \int_0^1 B_t^*(i)di = 0$), and combining with (66) and its foreign analog yields the world resource constraint,

$$Y_t P_{H,t} + Y_t^* P_{F,t}^* = P_t C_t + P_t^* C_t + P_{H,t} G_t + P_{F,t}^* G_t^* \tag{74}$$

4.9 Market Clearing

Having assumed that producers meet demand, market clearing in the domestic goods market is given by,

$$Y_t = \int_0^1 Y_t(i)di = \left(C_{H,t} + C_{H,t}^* + G_t\right) d_t \tag{75}$$

where,

$$d_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\eta} di$$

is the "measure of relative price distortion," (Yun 2004, p. 6) which can be solved recursively, along the same lines that delivered (61), to yield,

$$d_t = (1-\alpha)\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\eta} + \alpha\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{-\eta} d_{t-1} \tag{76}$$

By virtue of Walras's Law, the remaining market (the goods market in country F) will clear as all other markets clear (e.g. Romer (2001, p. 222)). It ought to be noted that there would be some measure of price dispersion (i.e. relative price distortion), analogous to (76), in the goods market clearing condition in country F; however, as Schmitt-Grohé and Uribe (2005, p. 18) point out, "studies that restrict attention to linear approximations to the equilibrium conditions are justified to ignore [d_t] if the model features no price dispersion in the deterministic steady state." As this will be the case here, whatever effects this price dispersion in the foreign goods market might have will be ignored. For the domestic goods market however, that this is indeed the case with proper initial conditions, will be shown.

4.10 Competitive Equilibrium

A competitive equilibrium is defined by the paths of the variables satisfying equations (4), (11), (12), (14), (16), the first equation in (18), (28), (29), (30), (40), (43), (44), (45), (48), (49) (58), (59), (60), (61), (62), (63), (64), (65), (74), (75), (76), the exogenous processes (22), (23), (53), (54), (67), (68), (70), (71), an interest setting rule for each country: (72) and (73), and a set of initial conditions.

5 The Model: Solution and Analysis

5.1 Solution Method

I shall solve the model using the Toolkit of Uhlig (1999) which entails a first-order Taylor approximation about a steady state.

The Steady State

The model will be linearized about a balanced-trade steady state, with steady-state consumption taxes and government expenditures given and prices and labor normalized[12].

With $\bar{\tau}^c$, $\bar{\tau}^{c*}$, $\bar{g}$, and $\bar{g}^*$ given, the steady-state relationships are:

$$\bar{R} = \bar{R}^* = \frac{1}{\beta} \tag{77}$$

$$\bar{c} = 1 - \bar{g} \tag{78}$$

$$\bar{c}^* = 1 - \bar{g}^* \tag{79}$$

$$\bar{\tau}^y = 1 - \bar{c}^{\sigma_c} \frac{\gamma}{\kappa_h} \frac{\eta - 1}{\eta} (1 + \bar{\tau}^c) \tag{80}$$

$$\bar{\tau}^{y*} = 1 - \bar{c}^{*\sigma_c} \frac{\gamma}{\kappa_h} \frac{\eta - 1}{\eta} (1 + \bar{\tau}^{c*}) \tag{81}$$

[12]See Appendix A.2 for derivation.

Examining (80) and (81), one sees that $\frac{(1-\bar{\tau}^y)}{(1+\bar{\tau}^c)} = \frac{(1-\bar{\tau}^{y*})}{(1+\bar{\tau}^{c*})}$, as $\bar{c} = \bar{c}^*$. If the initial conditions pertaining to the real exchange rate (51) are set to unity, then $\bar{\tau}^c = \bar{\tau}^{c*}$ and $\bar{\tau}^y = \bar{\tau}^{y*}$[13].

Log-Linearization

Uhlig (1999, pp.33-34) provides principles and "building blocks" relevant to the technique of log-linearization. Rather than review the process here, I shall simply state the definition, a rearrangement, an approximation, and a deviation from this technique used in this model.

The definition, for some variable X_t,

$$\hat{x}_t = ln(X_t) - ln(\bar{x})$$

exponentiating and rearranging gives,

$$X_t = \bar{x}e^{\hat{x}_t}$$

which can be approximated, assuming $\hat{x}_t$ is close to zero, by,

$$\bar{x}e^{\hat{x}_t} \approx \bar{x}\left(1 + \hat{x}_t\right)$$

I shall deviate from this technique regarding taxes. Whereas for all other variables this process (multiplied by one-hundred) yields percent-deviations from their respective steady states, this relationship is oft confusing for taxes. If one assumes a steady-state tax rate of say 34% percent, then a 1% increase in this tax rate from its steady state yields an increase of the tax rate by 0.34 percentage points. Such that taxes can be interpreted as percentage-point deviations from their respective steady-state rates, define,

$$\hat{x}_{t,redefined} = \hat{x}_t\bar{x}$$

where X_t is some tax rate. Thus,

$$X_t = \bar{x}e^{\hat{x}_t} \approx \bar{x}\left(1 + \hat{x}_t\right) = \bar{x}\left(1 + \frac{\hat{x}_{t,redefined}}{\bar{x}}\right)$$

In what follows, note that what I refer to as $\hat{\tau}_t^i$, $\forall i$ is actually $\hat{\tau}_t^i\bar{\tau}^i$, $\forall i$. Thus all log-linear taxes are to be interpreted as percentage-point deviations from their respective steady-state rates.

[13]See the discussion after (151) in Appendix A.2.

The Log-Linearized Model

The log-linearized model can be represented by the following equations, derivation provided in Section (A.3),

$$0 = -\hat{y}_t + (1-\theta)\bar{c}\hat{c}_t + \theta\bar{c}\hat{c}_t^* + 2\bar{c}(1-\theta)\omega\theta\hat{s}_t + \bar{g}\hat{g}_t \tag{82}$$

$$0 = -\hat{y}_t - \hat{y}_t^* + \bar{c}\hat{c}_t^* + \bar{c}\hat{c}_t + \bar{g}\hat{g}_t^* + \bar{g}\hat{g}_t \tag{83}$$

$$0 = -\hat{\pi}_t + \hat{\pi}_{H,t} + \theta\hat{s}_t - \theta\hat{s}_{t-1} \tag{84}$$

$$0 = -\hat{\pi}_t^* + \hat{\pi}_{F,t}^* - \theta\hat{s}_t + \theta\hat{s}_{t-1} \tag{85}$$

$$0 = \sigma_c\hat{c}_t - \sigma_c E_t\left[\hat{c}_{t+1}\right] - E_t\left[\hat{\pi}_{t+1}\right] + \frac{\hat{\tau}_t^c}{1+\bar{\tau}^c} - \frac{E_t\left[\hat{\tau}_{t+1}^c\right]}{1+\bar{\tau}^c} + \hat{R}_t \tag{86}$$

$$0 = \sigma_c\hat{c}_t^* - \sigma_c E_t\left[\hat{c}_{t+1}^*\right] - E_t\left[\hat{\pi}_{t+1}^*\right] + \frac{\hat{\tau}_t^{c*}}{1+\bar{\tau}^{c*}} - \frac{E_t\left[\hat{\tau}_{t+1}^{c*}\right]}{1+\bar{\tau}^{c*}} + \hat{R}_t^* \tag{87}$$

$$0 = -\hat{\pi}_{H,t} + \zeta_\pi\left[\zeta_y\hat{y}_t - \zeta_z\hat{z}_t + \sigma_c\hat{c}_t + \hat{s}_t + \frac{\hat{\tau}_t^c}{1+\bar{\tau}^c} + \frac{\hat{\tau}_t^y}{1-\bar{\tau}^y}\right] + \beta E_t\left[\hat{\pi}_{H,t+1}\right] \tag{88}$$

$$0 = -\hat{\pi}_{F,t}^* + \zeta_\pi\left[\zeta_y\hat{y}_t^* - \zeta_z\hat{z}_t^* + \sigma_c\hat{c}_t^* - \hat{s}_t + \frac{\hat{\tau}_t^{c*}}{1+\bar{\tau}^{c*}} + \frac{\hat{\tau}_t^{y*}}{1-\bar{\tau}^{y*}}\right] + \beta E_t\left[\hat{\pi}_{F,t+1}^*\right] \tag{89}$$

where, $\zeta_\pi = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$, $\zeta_y = \frac{1+\sigma_h-\gamma}{\gamma}$, $\zeta_z = \frac{1+\sigma_h}{\gamma}$

$$0 = E_t\left[\hat{e}_{t+1}\right] - \hat{e}_t + \hat{R}_t^* - \hat{R}_t \tag{90}$$

$$0 = -E_t\left[\hat{e}_{t+1}\right] + \hat{e}_t + E_t\left[\hat{s}_{t+1}\right] - \hat{s}_t + E_t\left[\hat{\pi}_{H,t+1}\right] - E_t\left[\hat{\pi}_{F,t+1}^*\right] \tag{91}$$

$$0 = -E_t\left[\hat{q}_{t+1}\right] + \hat{q}_t + E_t\left[\hat{\pi}_{t+1}^*\right] - E_t\left[\hat{\pi}_{t+1}\right] + E_t\left[\hat{e}_{t+1}\right] - \hat{e}_t \tag{92}$$

$$\bar{R}\hat{R}_t = \phi_y\hat{y}_t + \phi_\pi\hat{\pi}_t \tag{93}$$

$$\bar{R}^*\hat{R}_t^* = \phi_y^*\hat{y}_t^* + \phi_\pi^*\hat{\pi}_t^* \tag{94}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + \epsilon_t^z \tag{95}$$

$$\hat{z}_t^* = \rho_{z*}\hat{z}_{t-1}^* + \epsilon_t^{z*} \tag{96}$$

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \epsilon_t^g \tag{97}$$

$$\hat{g}_t^* = \rho_{g*}\hat{g}_{t-1}^* + \epsilon_t^{g*} \tag{98}$$

$$\hat{\tau}_t^y = \rho_\tau^y\hat{\tau}_{t-1}^y + \epsilon_t^{\tau,y} \tag{99}$$

$$\hat{\tau}_t^{y*} = \rho_\tau^{y*}\hat{\tau}_{t-1}^{y*} + \epsilon_t^{\tau,y*} \tag{100}$$

$$\hat{\tau}_t^c = \rho_\tau^c \hat{\tau}_{t-1}^c + \epsilon_t^{\tau,c} \tag{101}$$

$$\hat{\tau}_t^{c*} = \rho_\tau^{c*} \hat{\tau}_{t-1}^{c*} + \epsilon_t^{\tau,c*} \tag{102}$$

The only variables hitherto unexplained are $\hat{\pi}_{H,t}$ and $\hat{\pi}^*_{F,t}$. Gross producer price index (PPI) inflation in country H is defined, in log-linear form, as,

$$\hat{\pi}_{H,t} = \hat{p}_{H,t} - \hat{p}_{H,t-1}$$

and PPI inflation in country F is defined as,

$$\hat{\pi}^*_{F,t} = \hat{p}^*_{F,t} - \hat{p}^*_{F,t-1}$$

It is informative to note that in the equations above, CPI and PPI levels play no role in the dynamics of the model. It is only their gross rates of change, i.e. inflation, the are of consequence. Note that this is not a completely innocent method of solution: if the price levels are not determined, then neither is the nominal exchange rate. Thus in the following interpretations, it is the movements in the nominal exchange rate that are determined. Ljungqvist and Sargent (2004, pp. 872-880) discuss fiscal theories of the price level and their implications on exchange rate determinacy, but since the governments issue neither money nor debt in this model, I was unable to implement the suggestions presented there.

The aggregate demand curves for country H and country F are (86) and (87) respectively. Notice that (82) and (83) can be combined to deliver a log-linearized equation for the goods market in country F. This equation with (89) and (82) with (88) deliver New Keynesian Philips curves (i.e. aggregate supply supply curves) for countries F and H respectively, dependent upon future expectations, consumption in both countries, the terms of trade, and country specific shocks. Equations (90) and (91) can be combined to derive the real interest rate parity condition derived by Pappa (2002, p. 10). This is not done and (92), (84), and (85) are included to allow analysis of the effects of the exchange rates and to allow central banks to target CPI inflation.

Substituting the log-linearized variables into (1), the measure of welfare I shall use, gives,

$$U = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{\left(\bar{C} e^{\hat{C}_t}\right)^{1-\sigma_c}}{1-\sigma_c} - \kappa_h \frac{\left(\bar{H} e^{\hat{H}_t}\right)^{1+\sigma_h}}{1+\sigma_h} + \kappa_g \frac{\left(\bar{G} e^{\hat{G}_t}\right)^{1-\sigma_g}}{1-\sigma_g} \right) \right] \tag{103}$$

It ought to be noted that, "[...] the linearization method can generate inaccurate results in terms of welfare calculations, especially in open-economy models." (Kim and Kim 2004, p. 9) The issue at hand here is the following,

$$C_t = \bar{C} e^{\hat{C}_t}, \quad E_t \left[\bar{C} e^{\hat{C_{t+1}}} \right] = \bar{C} e^{\frac{\sigma_\epsilon^{c2}}{2}} \tag{104}$$

thus, I am using the unconditional mean instead of the conditional mean or the deterministic steady-state as opposed to the stochastic one which Kim and Kim (2003, pp. 477-479) shows to help prevent "spurious welfare reversals." For example, with the uncovered interest rate parity condition (48), this model exhibits a consumption risk sharing mechanism. As this will tend to reduce the variance of consumption, we see that the equation above predicts that expected consumption without risk sharing would be higher than with risk sharing: a spurious suggestion. I shall leave corrections of this sort to future investigation but shall be watchful for results consistent with welfare reversals.

5.2 Calibration

Table 1 gives the values and sources for the specification I use in this model. β is the subjective discount factor, assumed to be common across countries, and yields a steady-state annual interest rate of $0.99^{-4} = 1.041$ or about four percent net per annum. σ_c is the constant of relative risk aversion, this parameter varies in the literature, but values between one and six seem to be the most common. Regarding σ_h or the inverse of the Frisch elasticity of labor, there is no common stance in the literature: Schmitt-Grohé and Uribe (2005, p. 23) use 4, Pappa (2002, p. 14) says that empirics support values between one and ten, while Canzoneri, Cumby, and Diba (2005, p. 45) claim empirical support for values between 0.35^{-1} and 20. The value I choose, three, fits well amongst these differing specifications. σ_g and κ_g are assumed to take the values two and one respectively; the former sets the elasticity of utility with respect to government purchases equal to that regarding consumption and the latter ensures that government expenditures have equal weight in households' utility. Though one could certainly argue that this is not the case in reality, this simplifies the setup.

ω and η, the elasticity of substitution between international goods and intranational goods respectively, are set to one and a half and six, respectively.

Table 1: Parameter Calibration

Parameter	Value	Source
β	0.99	Bachus, Kehoe, and Kydland (1992, p. 758)
σ_c	2	Pappa (2002, p. 28)
σ_h	3	(see discussion below)
σ_g	2	Assumed for simplicity
ω	1.5	Pappa (2002, p. 28)
η	6	Schmitt-Grohé and Uribe (2005, p. 26)
α	0.75	Pappa (2002, p. 28)
γ	1	e.g. Galí and Monacelli (2002, p. 7)
κ_g	1	Assumed for simplicity
κ_h	1	e.g. Galí and Monacelli (2002, p. 4)
$\rho_z = \rho_{z*}$	0.9	Bachus, Kehoe, and Kydland (1992, p. 760)
$\rho_g = \rho_{g*}$	0.9	Canzoneri, Cumby, and Diba (2005, p. 15)
$\rho_{\tau_c} = \rho_{\tau^{c*}}$	0.85	Canzoneri, Cumby, and Diba (2005, p. 16) Kim and Kim (2004, p. 8)
$\rho_{\tau^y} = \rho_{\tau^{y*}}$	0.85	Canzoneri, Cumby, and Diba (2005, p. 16) Kim and Kim (2004, p. 8)
$\sigma_\epsilon^z = \sigma_{\epsilon^z}^*$	0.00852	Bachus, Kehoe, and Kydland (1992, p. 760)
$\sigma_\epsilon^g = \sigma_{\epsilon^g}^*$	0.015	Canzoneri, Cumby, and Diba (2005, p. 15)
$\sigma_\epsilon^{\tau_c} = \sigma_\epsilon^{\tau_c^*}$	0.014	Kim and Kim (2004, p. 8)
$\sigma_\epsilon^{\tau_y} = \sigma_\epsilon^{\tau_y^*}$	0.05	Kim and Kim (2004, p. 8)
$\bar{\tau}_c = \bar{\tau}_c^*$	15%	Canzoneri, Cumby, and Diba (2005, p. 14)
$\bar{g} = \bar{g}^*$	0.22	Canzoneri, Cumby, and Diba (2005, p. 14)

Much of the literature sets the former to one, which would have simplified the foregoing calculations, but as discussed previously, would neglect important international spill-over effects. α, the probability of not being able to adjust prices, implies an average contract length of three quarters and is the most frequently used value I came across. γ being equal to one implies a production function with constant returns to scale. κ_h is set to one along the same logic as κ_g, this is standard in most models. ρ_i for all the shocks measures their autocorrelation and σ_ϵ^i their standard deviations.

Kim and Kim (2004, p. 8) note, "[m]easuring aggregate tax rates is a complex and difficult task and there is little consensus on effective tax rate measures," and report average consumption tax rates for the G-7 between 5% (Japan) and 20% (France) with a G-7-wide average of 12% with a persistence measure of 0.084 and a standard deviation of 0.014; Canzoneri, Cumby, and

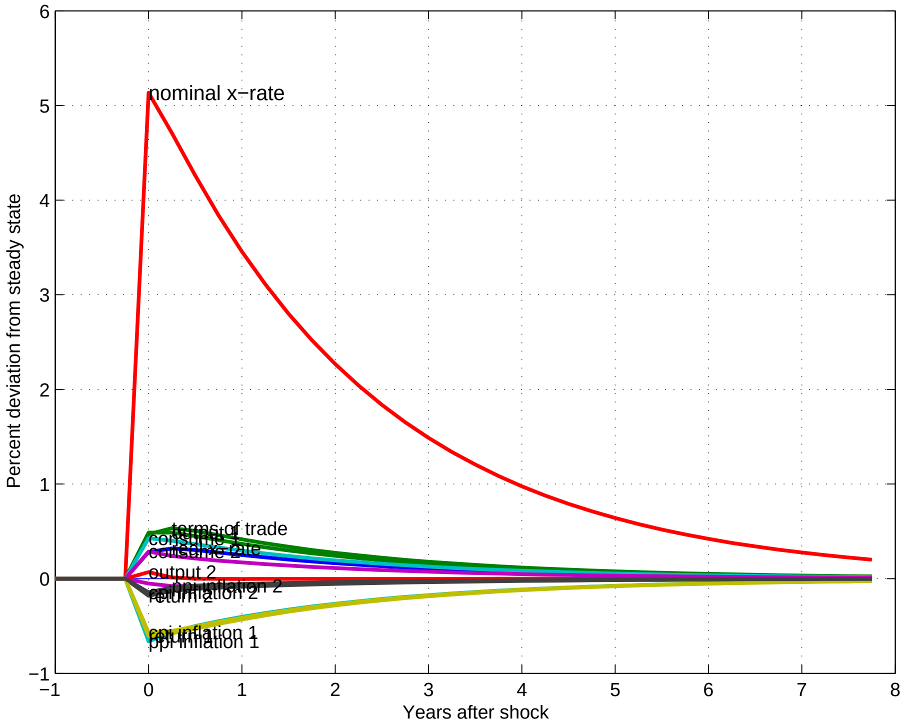
Diba (2005, p. 14) use steady-state consumption tax levels of 15% and 17% for high- and low-debt countries, respectively, computed as averages for EU countries from 1995-2001; and Herz, Roeger, and Vogel (2004, p. 12) use a value of 20% obtained from an un-weighted average of VAT rates from a selection of smaller European countries in their small open-economy model. My values for the autocorrelation, standard deviation, and steady state levels of consumption taxes falls within these guidelines. I let the steady-state levels of production taxes be solved for endogenously, see (80) and (81), and set their autocorrelation parameters and standard deviations to values in the range presented by Kim and Kim (2004) for capital and wage tax rates.

5.3 Baseline Impulse Responses

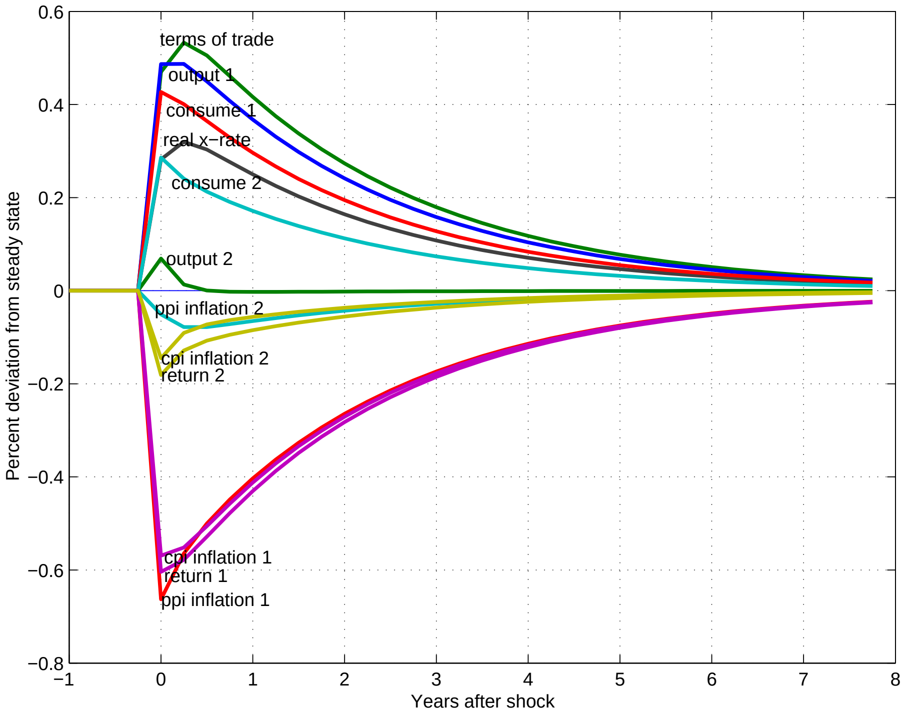
Domestic Technology Shock

Figure 1 shows the impulse responses to a one-percent deviation in technology. Notably, Figure 1(a) shows the large deviation in the nominal exchange rate. Though one might consider this to be an indicator of "excess volatility" (depending upon the qualification of the term "excess"), there is no "disconnect" in the nominal exchange rate here (and certainly not in the real exchange rate): the movements in the exchange rates are generated by movements in other macroeconomic variables. Recalling my note about exchange rate determination earlier, it is important to remember that only exchange rate movements are to be interpreted. In alternative solutions to this model, I found this impulse response, sunspots, or an exchange rate deviation from its steady state with the same movements but starting from the x-axis and ending at a new steady state equal to the negative of its initial response here. The logic is simple: as Pappa (2002, p. 7) notes, purchasing power parity will not hold if the international elasticity of substitution is not equal to one; thus, we either begin where the exchange rate is equal to its steady state or we end up there (which is what happens here by using expectations in (91) and (92)), but not both.

Firstly, notice that output in the domestic country increases by about one-half of one percent, consumption in both countries increases substantially (though more in the domestic country than in the foreign), and finally that output in the foreign country increases initially before falling to barely below its steady-state value. That a technology shock at home should increase



(a) All Variables



(b) Selected Variables

Figure 1: Baseline Impulse Response to a One-Percent Deviation in Technology

Exogenous Taxes

output at home, ought not to come as a surprise: as production becomes more effective, more ought to (and is) produced. Interesting is that output increase by less than the productivity shock: Pappa (2002, p. 17) explains this through an income effect. The same holds here: examining the production function, (52), it is easy to see, recalling that $\gamma = 1$, that if productivity increases by less than the productivity shock, hours worked H_t must decrease, thus households in country H consume more but work less.

As consumers are producers as well, they take their consumption into account when setting prices. As international risk sharing increases the consumption of households in both countries, those producers who can change their prices would like to raise them, as else being equal. In the domestic country, the effect of productivity shock dominates the combined increase in consumption, output, and the terms of trade, causing a deflation in the producer prices of country H, tempered by expectations of future deflation. In the country F, the inflationary pressure from increased consumption is counteracted by the terms of trade, causing a mild deflation in the producer prices of country F; once output falls past zero, inflation begins to subside.

As the terms of trade (the ratio of the price of foreign goods to that of domestic goods in country H) rise, consumers in both countries engage in consumption switching and purchase more goods produced in country H. Notice that the terms of trade only begin to fall after the first quarter. This fall only begins after PPI inflation in each country is equal to CPI inflation in the respective country. For example, in country H until the first quarter, domestic producer prices are falling more quickly the consumer prices, implying that foreign producer prices denominated in the home currency do not fall as quickly as domestic producer prices. From the second quarter on, the CPI inflation is lower than PPI inflation in country H, implying that the price for foreign goods in country H falls more quickly than the price for domestic goods.

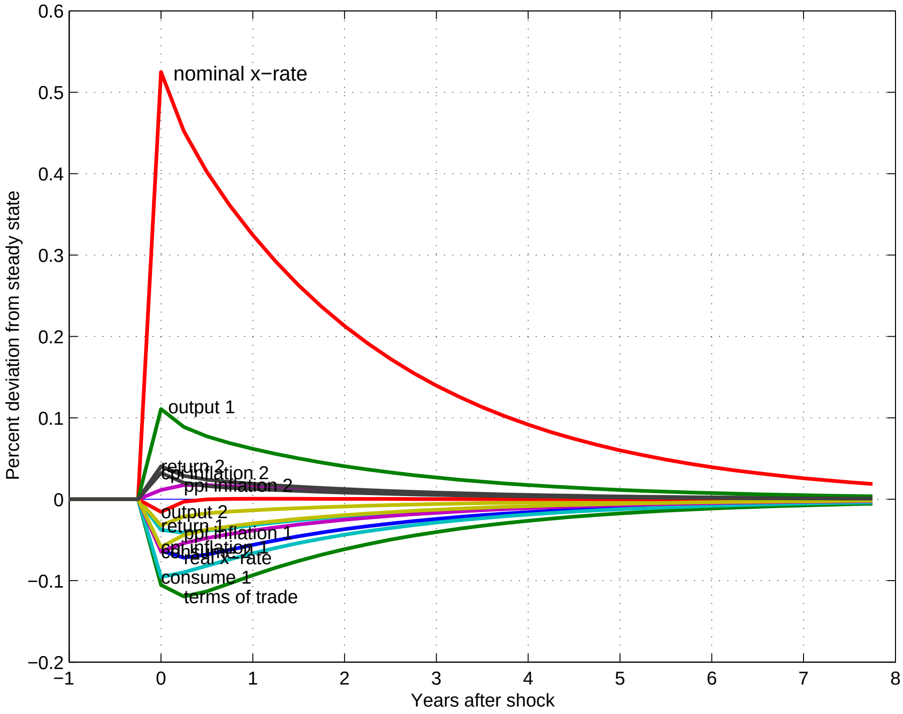
That output in the foreign country should increase in response to a domestic productivity shock is slightly perplexing and certainly not in line with the impulse responses one generally sees in the open-economy NKM's. One would expect that both domestic and foreign households should engage in consumption switching to such an extent that demand for foreign goods falls, which, cæteris paribus, should decrease output in country F. That this does not happen is partly due to the observation that monetary authorities, as previously mentioned, ought to take movements in the real exchange rate into account. In that vein, the central bank in country F reduces the nominal interest rate by too much, artificially increasing output: notice that when CPI and PPI inflation intersect, output in the foreign country falls to zero. PPI inflation in country F is forced to decrease until the terms of trade starts to decline. Alternatively, monetary authorities could target PPI inflation instead of CPI inflation, thereby focusing on the country-specific origin of inflation, this would lead to higher output in country H and lower in country F than what is seen here.

Domestic Government Expenditures

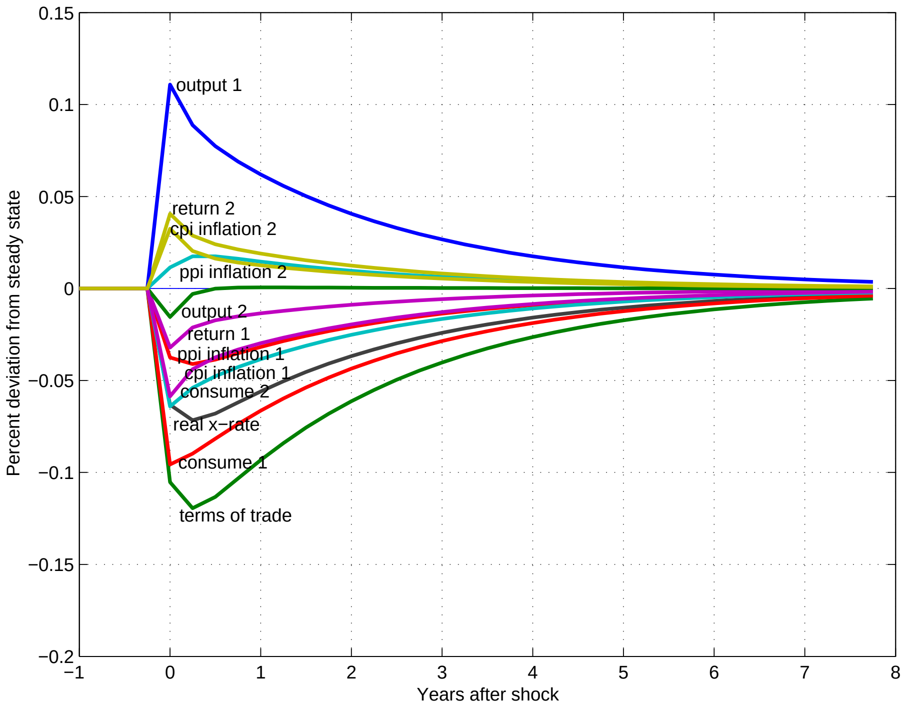
Figure 2 shows the impulse response to a one-percent increase in government expenditures. Once again, the nominal exchange rate displays a more severe deviation from its steady state than other variables, though, once again, this is not disconnected from the model's fundamentals. Notice that output in the domestic country increases, consumption decreases, and a deflation occurs. As governments are assumed to consume only those goods produced in their respective countries, an increase in the consumption of the government in country H increases the output in country H, all else being equal. That domestic consumption falls is due to a crowding-out effect. Canzoneri, Cumby, and Diba (2005, p. 39), however, note that VAR-analyses have shown that government purchases do not crowd out consumption and consider this to be a major setback of the models of this type.

Through consumption risk sharing, households in country F will also face decreased consumption. The terms of trade fall, meaning that goods produced in country F are relatively more inexpensive than those produced in country H, thus households switch consumption to goods produced in country F. Output in country F also deviates negatively from its steady state, implying that the consumption switching effect is outweighed by crowding out and risk sharing. Though total demand increases for goods in country H, producers lower their prices as the decrease in domestic consumption and the terms of trade outweigh outweigh the increase in output. PPI inflation in country F is positive and hump-shaped: the terms of trade continue to fall into the second quarter, surpassing the increase in foreign output and consumption experienced after their initial falls.

Once again, by acting upon CPI inflation and output, central banks in



(a) All Variables



(b) Selected Variables

Figure 2: Baseline Impulse Response to a One-Percent Deviation in Government Expenditures

Exogenous Taxes

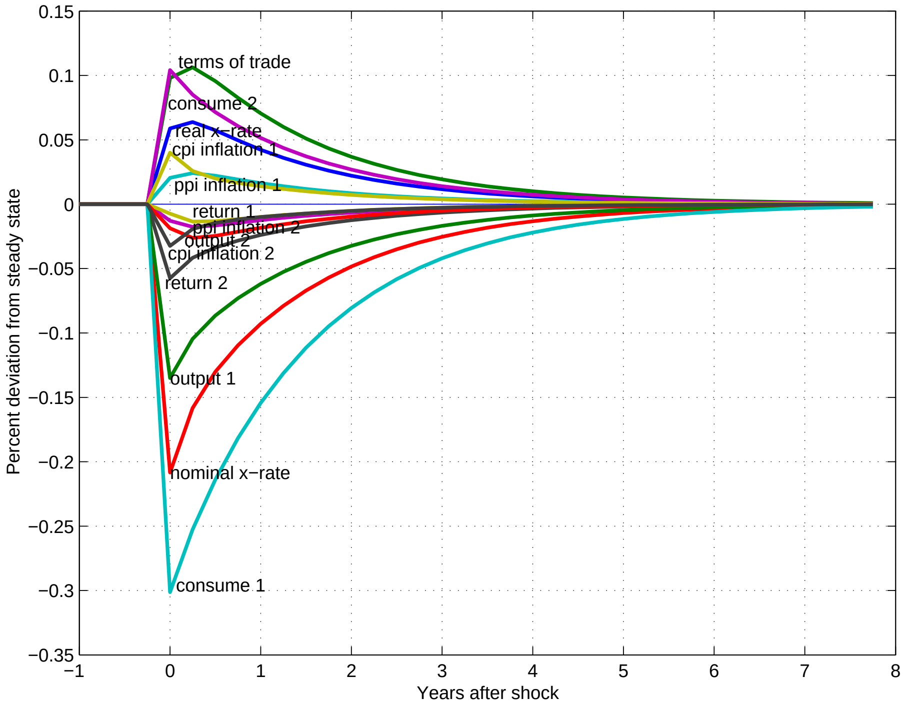


Figure 3: Baseline Impulse Response to a One-Percentage-Point Increase in Consumption Taxes

Exogenous Taxes

both countries choose interest rates that are too high in the first two quarters; thus, the effect observed in country F's output may be in part due to constrictive monetary policy on the part of its central bank. Notice that foreign output once again crosses the x-axis when PPI and CPI inflation in country F are equal. Likewise, output in country H would rise initially to a higher deviation from its steady-state value but fall more swiftly, were the return in country H to respond to PPI inflation instead of CPI inflation.

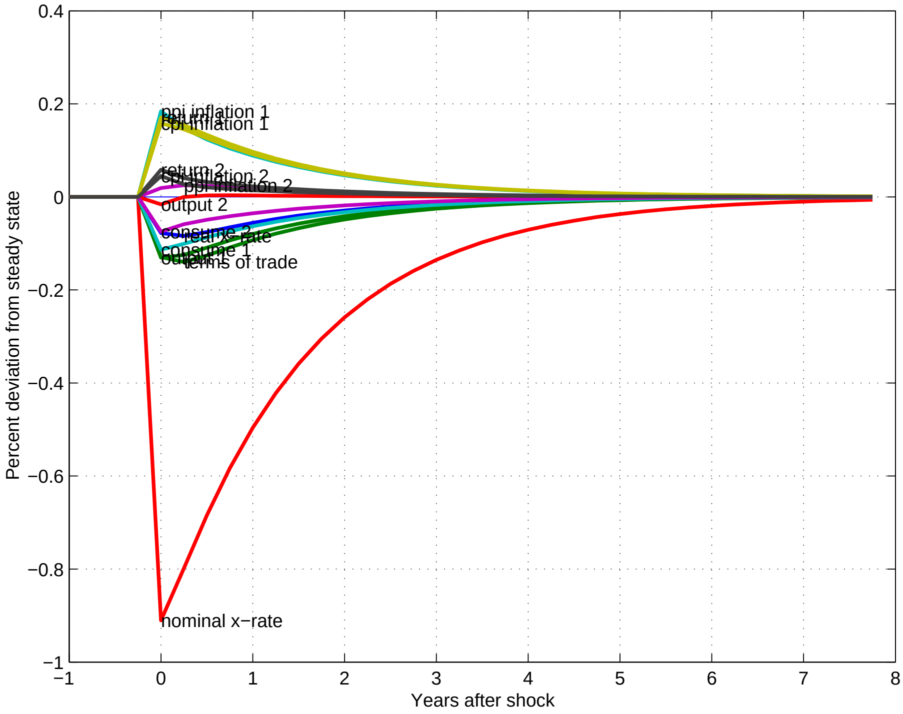
Domestic Consumption Tax

Figure 3 shows the impulse response of the model to a deviation in the tax on consumption in country H by one percentage point from its steady-state rate, recall the manner in which deviations in tax rates were defined earlier. The increase in consumption taxes reduces consumption in country H and puts inflationary pressure on producer prices in the same. The terms of trade increase, as consumers engage in consumption switching, consuming relatively more of the goods produced in country H than in country F. As before, the terms of trade begin to decrease when PPI inflation is higher than CPI inflation in country H. The responses of the interest rates, like before, are predicated on output and PPI inflation in their respective countries.

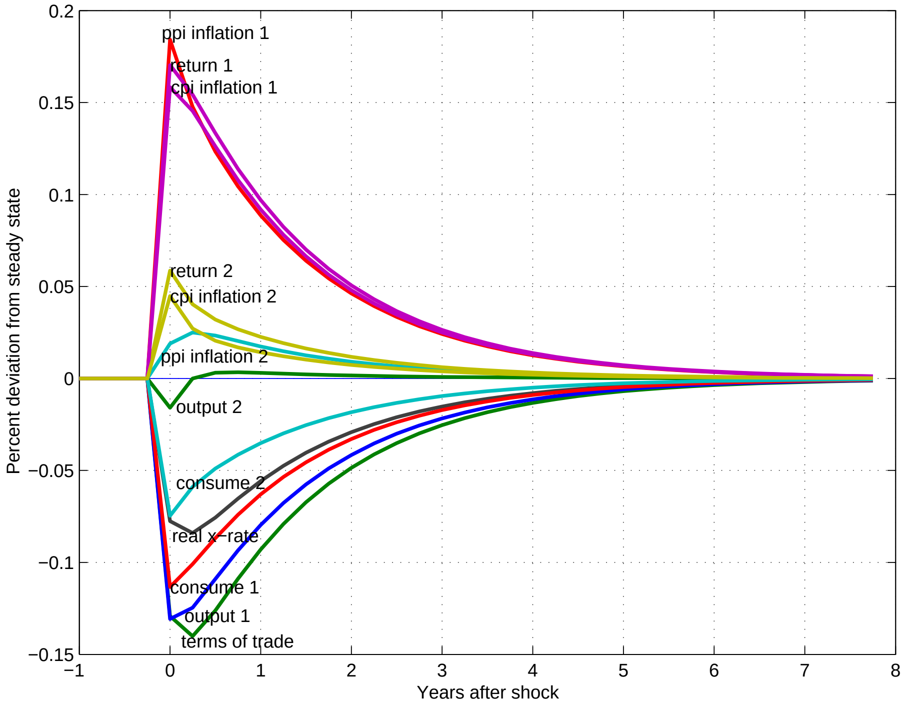
Notice that consumption in country F increases, while output in both countries and consumption in country H decrease. This is a powerful result: consumption taxes drive a wedge in the consumption-risk-sharing effect. This can be seen in (50), where marginal *net of consumption tax* consumption in both countries is equalized through the real exchange rate. Alternatively, as the real exchange rate is driven through the uncovered interest rate parity condition and thus intertemporal consumption decisions: the higher rate of consumption tax today is accompanied by the expectation of relatively lower consumption taxes in the future, and, thus, consumers in country H will switch consumption intertemporally.

Domestic Production Tax

A positive one-percentage-point deviation of the production tax in country H causes inflation and a decrease in the output of country H from its steady state. As the tax on production works through marginal costs, it works very much like a negative deviation in technology, at magnitude reduced by approximately one-fifth. Its persistence is, however, less than that of technology, which is why output in country H, for example, increases throughout from its initial fall as opposed to the steady deviation observed from the time of the technology shock till the following period. There is still, however, a noticeable change in its slope after the end of the first quarter due to the the response of the interest rate to PPI inflation. This effect can be most easily seen as occurring when the rates of PPI and CPI inflation intersect.



(a) All Variables



(b) Selected Variables

Figure 4: Baseline Impulse Response to a One-Percentage-Point Increase in Production Taxes

Exogenous Taxes

6 Extending to Endogenous Fiscal Policy

6.1 Policy Rules

In this section, I shall evaluate tax policies in my model by means of a parameter search. In order to search for parameter values, I need policy rules for whose parameters I can search. Kim and Kim (2004, p. 6-7) use,

$$\tau_t = \bar{\tau} + \eta A_t \tag{105}$$

where A_t is the productivity shock. If output is highly correlated with the productivity shock, which seems very likely given that the former is a function of the latter, η determines not only the sensitivity of taxes but also the their cyclicity. Canzoneri, Cumby, and Diba (2005, pp. 25-26) use,

$$\tau_{w,t} = \bar{\tau}_w - \zeta_w E_{t-1}\left[\frac{S_t}{Y_t}\right] + \epsilon_{\tau w,t} \tag{106}$$

where S_t is the real budget surplus. A very useful observation here is that taxes respond with a one period delay, that is, taxes today are a function of yesterday's information. This is based on the assumption that, "the legislative process is too sluggish for the fiscal authorities to react to deficit/GDP within the quarter, and that they determine purchases and tax rates one period in advance." (Canzoneri, Cumby, and Diba 2005, p. 25) Herz, Roeger, and Vogel (2004, p. 11) use,

$$\Delta\tau_t = \psi_x X_t + \psi_\pi \pi_{H,t} \tag{107}$$

where $\Delta\tau_t = \Delta\tau_{t-1}$ and X_t is the output gap (the deviation of output from the level it would have obtained in the absence of nominal rigidities). Once again, having noticed the timing of taxes, that fiscal authorities respond with a delay, or respond to past circumstances, in determining taxation, see above. Schmitt-Grohé and Uribe (2005, pp.38-39) use,

$$\tau_t - \tau^* = \beta_a ln(\frac{a_t}{a^*}) + \beta_y ln(\frac{y_t}{y^*}) + \beta_\tau ln(\tau_{t-1} - \tau^*) \tag{108}$$

where a_t is the productivity shock and an asterisk denotes the Ramsey steady-state value. Here, taxes are smoothed over time, with β_τ being the smoothing parameter. Note that an extension incorporates "time to tax," noting that, " it is unrealistic to assume that tax rates change every quarter."(Schmitt-Grohé and Uribe 2005, p. 45) In this extension, tax rates are modified by lags to force taxes to be determined several quarters in advance.

I shall proceed as follows: as the analysis by Kim and Kim (2004, p. 7), setting taxes as a function of current productivity, allows one to readily examine the cyclicity of taxes. Though I could use $\hat{y}_t$ as a feedback parameter, and indeed shall in the second policy rule, using the productivity shock $\hat{z}_t$ will better facilitate comparison with the aforementioned authors' results. I will, however, adjust the policy rule to take into account that taxes ought to be implemented with at least a minimal delay to capture the intuition that fiscal policy makers cannot respond as quickly as monetary policy makers Thus, the first fiscal policy rule will be,

$$\hat{\tau}_t^i = \phi_\tau^i E_{t-1}\left[\hat{z}_t\right], \ i = c, y \tag{109}$$

This policy rule is, however, very limited. Noticing that Kim and Kim (2004, p. 6) hold government purchases constant, eliminating a demand shock present in my model, it might be wise to compare this policy rule with one that would include some feedback from a demand shock.

Two of the proceeding four policy rules have output in the feedback rule, in contrast with the foregoing rule, this allows demand shocks to enter into the analysis. The second tax policy, similar to that of Canzoneri, Cumby, and Diba (2005, pp. 25-26), though government surpluses are omitted as they do not exist by assumption in my model, essentially allows me to examine the validity of the claim of Kim and Kim (2004) that similar results hold regardless of whether productivity shocks or output is used as the feedback parameter in this setting and is given (in log-linearized form) by,

$$\hat{\tau}_t^i = \phi_\tau^i E_{t-1}\left[\hat{y}_t\right], \ i = c, y \tag{110}$$

Simple intuition would compel one to anticipate that the parameters here ought to take different values than in the previous rule. As demand shocks are present in this model, targeting output involves substantially more input than targeting simply productivity shocks. Furthermore, feedback from output will include international spill-over effects.

Two observations are in order before I proceed. First, I shall abstract from policy shocks, though this implies that fiscal policy makers follow a rule mechanically (in all likelihood not very realistic, but this was assumed for monetary policy as well), it does not affect the impulse responses to the productivity and government expenditures shocks – this does, however, affect the variances derived from simulations, but I will leave this to further investigation. Second, I have assumed no tax smoothing, Schmitt-Grohé and Uribe (2005, pp.38-39) find that this parameter is significantly different from zero (about 0.3) in their model, but as I have abstracted from this feature in monetary policy (Clarida, Galí, and Gertler (1998) find, in an empirical study, that this smoothing parameter is relatively high at about 0.9 for all all countries studied); I feel it would not make any sense to smooth one but not the other: comparing the effects of different amounts of smoothing in fiscal and monetary policies could be a very interesting area of future investigation.

6.2 One-Sided Endogenous Policy Parameter Decisions

I will run simulations of the economy and use (103) to calculate utility for the consumer. In running simulations, a random trajectory for all the shocks will be used; i.e., I run the Toolkit program of Uhlig (1999) once, save the values for "sim eps", the randomly generated shocks for simulations, and "sim discard eps", the randomly generated values of shocks discarded at the beginning of simulations, and use these values in simulations for all parameter combinations examined. As results may be specific to a particular draw, I will take fifty random draws of shock combinations, average the utility from each specification over the the fifty draws, and then determine which parameter set maximizes the consumer utility over the average of the draws. I reduce the length of simulations to 36 quarters (translating to nine years), still a long horizon for policy makers to be concerned with. The shortening of the length of simulations (the default is 100 quarters) and limiting myself to fifty draws is done limit the time needed to perform calculations: MATLAB[14] needs approximately six one-hundredths of a second per simulation on my computer, with two parameters being allowed to take twenty values each and this being repeated over the fifty draws, I need approximately twenty minutes to generate and analyze the payoffs generated by these combinations. Though not time-prohibitive, calculations for Nash and cooperative equilibria, with both policy makers choosing simultaneously, require five and a half days with the same specifications, I will discuss the ramifications of this issue in the following section. Although a greater number of draws would certainly increase the chances that the results are not draw-specific, I believe fifty to

[14]MATLAB is a registered trademark of The Math Works, Inc., hereinafter not noted.

be a good balance between accuracy and time-efficiency.

In my first run, I shall maintain one of the countries' taxes as exogenous processes and search for those parameter values that maximize the discounted utility of that country's representative household whose fiscal policy is being endogenized. Thus, the domestic fiscal authority will maximize the discounted utility of the domestic representative consumer, (103), taking the foreign fiscal policy as exogenous and then the foreign fiscal authority will maximize the discounted utility of the foreign representative consumer, foreign analog to (103), taking the domestic fiscal policy as exogenous. For welfare comparisons, the utility of households with the baseline specification, i.e. exogenous fiscal policy, over the same horizon is -192.15 for country H and -193.02 for country F. As with all the results obtained hereafter, these numbers are influenced by the exact draws for the simulations; as the number of draws approaches infinity, there should be no variation in this number. As discussed in the proceeding paragraph, however, I shall limit myself to fifty draws and as such, these numbers may not always be the same for a repetition of a given experiment. With fifty draws, these numbers, though, ought to not deviate too extensively from those given here.

Using the first fiscal rule (109), I maximize the domestic household's utility using the interval $\phi_\tau^i = [-9.5, 9.5]$ in steps of one holding foreign fiscal policy exogenous. Then, I repeat the process on the same interval with foreign fiscal policy being examined, holding domestic fiscal policy exogenous. The results are presented in Table 2. Most striking is the fact that all parameters are positive. Though I did not include zero as a possible parameter choice, that both parameters for country F took values as close to zero as pos-

Table 2: Single Country Endogenized Feedback Rules, Specification (109)

	Endog. Domestic Policy Exog. Foreign Policy	Endog. Foreign Policy Exog. Domestic Policy
ϕ_τ^c	5.5	—
ϕ_τ^y	2.5	—
ϕ_τ^{c*}	—	0.5
ϕ_τ^{y*}	—	0.5
U	-191.69	-194.05
U^*	-200.34	-192.98

sible could lend itself to the conclusion that active fiscal policy is not welfare improving for country F. Examining the values for welfare and comparing them with the baseline values for utility, −192.15 for country H and −193.02 for country F, one sees that active fiscal policy is, however, welfare improving for both country H and country F. Using the definition given by Kim and Kim (2004, p. 7), these rules recommend countercyclical fiscal policy. If the other country's fiscal policy is exogenous, households in each country would be better off if their respective policy makers increased taxed in response to a domestic technology shock. It should be noted that the welfare gains are small: two-tenths of one percent for country H and two-hundredths of one percent for country F.

Why should one country react far more strongly than the other? In my opinion, this is a sort of fiscal adjustment to monetary policy. Recalling (50), one sees that consumption taxes in both countries, all else being equal, will have opposite effects on the real exchange rate. If one of the two monetary authorities ought to be considering real exchange rate movements, as noted before as a possibility, the asymmetrical results could be explained here. One might be able to conclude that if monetary policy were to react to country-specific inflation, or to compensate by using the real exchange rate, optimal taxes would converge for consumption and production in the two countries.

The optimal parameter values for taxes using the policy rule (110) shows rather interesting results, summarized in Table 3. First off, country H should raise taxes in times of lower output and lower them to accommodate increased output. Fiscal policy in country F, however, should be mixed: increasing taxes on production and decreasing those on consumption when output de-

Table 3: Single Country Endogenized Feedback Rules, Specification (110)

	Endog. Domestic Policy Exog. Foreign Policy	Endog. Foreign Policy Exog. Domestic Policy
ϕ_τ^c	-2.5	—
ϕ_τ^y	-1.5	—
ϕ_τ^{c*}	—	-4.5
ϕ_τ^{y*}	—	8.5
U	-191.21	-188.74
U^*	-188.29	-189.31

viates positively from its steady state. Rather perplexing is the result that in each case, that country's consumers whose fiscal policy was held exogenous reaped the greater utility gains.

Considering the policy of country F, lowering the tax rate on consumption and raising the tax rate on consumption has several features that coincide, one of which is that they will reduce consumption in country H. The effects on output and consumption in country F are ambiguous, however output in country H should tend to increase. This would seem to me to lead to lower utility in country H, less consumption and more output (i.e. in the absence of a technology shock more hours worked). Yet the effect on the utility of households in country H in response to foreign fiscal policy setting in Table 3 contradict the begger-thy-neighbor effect that would seem to take place. Though with higher interest rates, households in country H could benefit by intertemporally shifting consumption, mitigating this apparent contradiction.

6.3 Nash Games between Fiscal Policy Makers

In this section, fiscal policy makers will choose values for the parameters in their fiscal policy rules simultaneously. Given a particular parameter combination of the policy maker in country F, the strategy of the policy maker in country H will be that parameter combination that maximizes the utility of the household in country H. The same for the other way around: given a combination of parameter values for the policy rule in country H, the strategy of the policy maker in country F will be that parameter combination for his policy rule which maximizes the utility of the household in country F.

A Nash equilibrium will a combination of parameter values for the policy rule in country H and those for the policy rule in country F, whereby neither policy maker has an incentive to deviate from his parameter value combination. That is, a Nash equilibrium will be that combination of parameter values for the policy rule in country F for which the policy maker in country H delivers a combination of parameter values that do not induce the policy maker in country F to deviate from his aforesaid combination, and vice versa.

Using a similar procedure as before, I shall generate payoff matrices for country H and for country F. Each item in the payoff matrices corresponds to the utility for the household in the country of the respective payoff matrix.

Nash equilibria (or the Nash equilibrium as the case my be) will be calculated by a simple program [15] that defines a Nash equilibrium as a pair of strategies for countries H and F, say ‘i’ and ‘j’, for which ‘i’ yields the greatest utility for country H given ‘j’ and ’j’ yields the greatest utility for country F given ‘i’ [16].

Whereas twenty values per parameter were allowed in the derivation of optimal policies given that the other country’s policy is exogenous, I limit the number of possible values per parameter to ten in computing the policy games. Note that with four parameters, twenty values per parameter, and fifty draws, I would need to run approximately eight million simulations (approximately as for each draw an additional simulation is necessary to produce the trajectory of the exogenous processes used in the simulation of each alternate parameter specification). To generate and analyze the payoff matrices, my computer would need five and a half days. As I need to be able to make adjustments to the programs, as it turns out I made plenty of typographical and logic mistakes along the way, the reduction of possible parameter values to ten reduces the needed time to just over eight hours. I should hope to find more efficient programs to facilitate my calculations in the future, but for my current needs, I feel this to be more than justifiable. In choosing the range of parameter values, I reduce the range to [-4.5, 4.5]; this does not allow all the parameter values found above, but allows for both highly sensitive responses (greater than one in absolute value) and muted responses (less than one in absolute value).

6.4 Fiscal Policy Interaction, Rule (109)

Table 4 summarizes the results from the policy games played by the fiscal policy makers in countries H and F using the policy rule given in equation (109). There exists a unique Nash equilibrium with the given range of parameter values for the policy rules. I have divided the results into the Nash equilibrium, the best parameter combinations (regardless of stability) for each country, and the best cooperative solution. Of the possible ten thou-

[15] I use a version of K. Passino’s program available for download at http://www.ece.osu.edu/~passino/ICbook/Code/nashequilibria.m that I have extensively modified for my uses here.

[16] This is done in MATLAB with a nested loop and an inequality condition $J1(i, j) \geq \max(J1(:, j))$ and $J2(i, j) \geq \max(J2(i, :))$, where $J1$ and $J2$ are the payoff matrices for countries H and F respectively.

Table 4: Fiscal Policy Games, Specification (109)

	Nash	Best (H)	Best (F)	Best Coop
ϕ_τ^c	1.5	1.5	-0.5	-0.5
ϕ_τ^y	0.5	0.5	-0.5	-0.5
ϕ_τ^{c*}	3.5	0.5	3.5	-0.5
ϕ_τ^{y*}	1.5	0.5	1.5	-0.5
U	-196.87	-191.87	-196.94	-192.45
U^*	-194.22	-195.62	-193.2	-193.49

sand parameter combinations, 101 yield a higher world utility than the Nash equilibrium. I have presented only that cooperative solution which yields the highest world utility amongst all cooperative solutions.

Perhaps the most perplexing observation is that only one of the results yield a utility for either country higher than the baseline level under exogenous taxes, -192.15 for country H and -193.02 for country F. A possible conclusion would be that if the option of letting taxes follow a random walk had been included, this would have been the optimal cooperative solution. This would certainly be in line with Schmitt-Grohé and Uribe (2004, pp. 216-7), who find that taxes, derived by a Ramsey policy maker in a closed economy, should follow such a random walk. I am a little hesitant to jump to this conclusion, noting that the utility for both countries is higher when all parameters are set to -0.5, as in the best cooperative solution, than when all parameters are set to 0.5, one of the 100 other cooperative solutions. This makes me think that values very close to zero, but still negative might have resulted in a cooperative equilibrium that yields a higher amount of utility for both countries than in the baseline specification.

The Nash equilibrium calls for taxes in each country to deviate positively from their respective steady states in response to a positive deviation of technology from its steady state in the same country. Interestingly, the parameter values in the Nash equilibrium correspond to the values that each country would need to pick in order to obtain the highest level of utility for its households.

The parameter combinations which yield the highest utility for country H and those which yield the highest utility for country F, Best (H) and Best (F) respectively, serve to reinforce the ideal of an imbalance that fiscal policy ought to try and correct. These combinations are not stable in the sense that the policy maker in country F would not choose the parameter combination indicated in Best (H) given the policy of country H and vice versa.

Comparing the results to Table 2, it is interesting to note that, under the Nash equilibrium, country F's tax rates should be more sensitive to the feedback variables than those of country H. This is exactly the opposite of the result in Table 2. Notice, however, that the ideal combination for country H would still have its consumption tax reacting more strongly to the its productivity shock than should country F's.

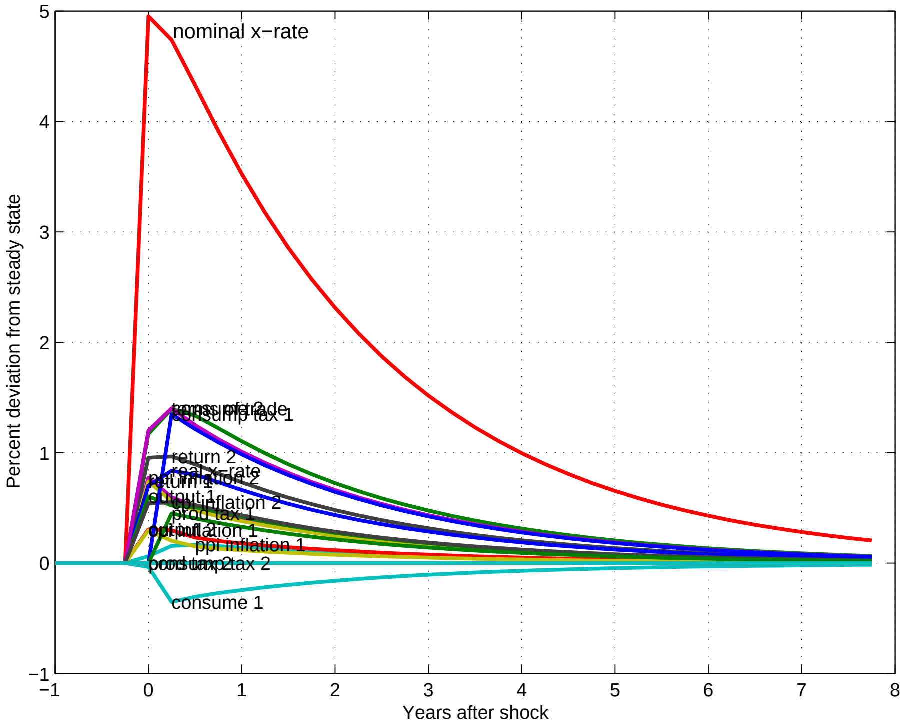
Regarding taxes dependant upon the technology shock, we can conclude that cooperation is beneficial. If fiscal authorities cooperate and let their taxes follow a random walk, or a policy very close one, gains to society are higher than under the Nash equilibrium, where the strategic actions of fiscal authorities lead to reduced welfare in both countries.

Impulse Response to a One-Percent Deviation in Domestic Technology in the Nash Equilibrium

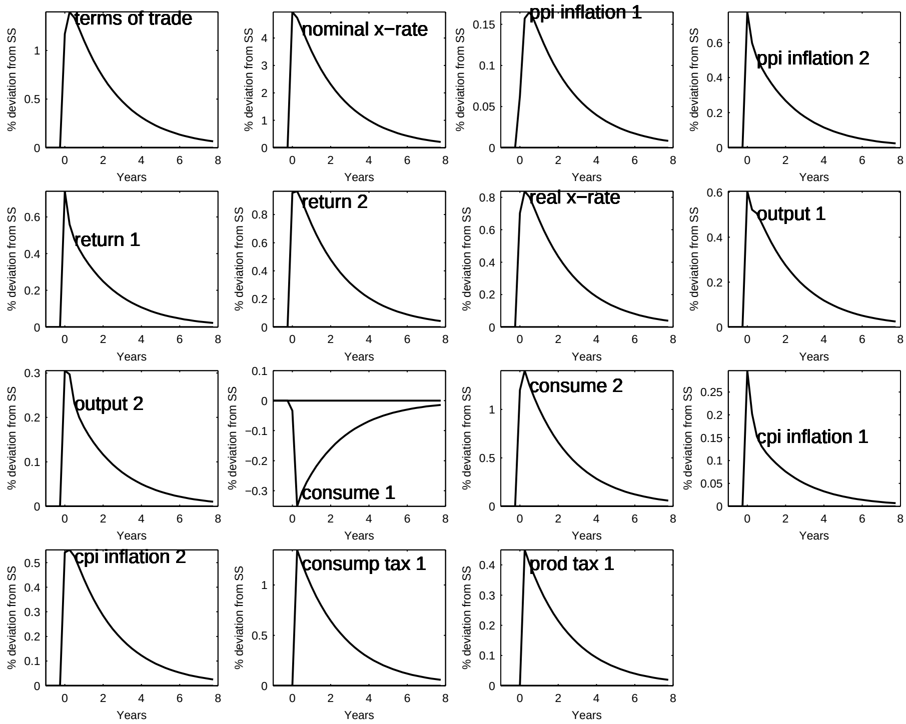
Figure 5 shows the impulse response to a domestic technology shock of the system with endogenous taxes according to the Nash equilibrium in Table 4. According to the policy rule, the fiscal policy maker in country H will raise the tax on consumption and on production by 1.5% and 0.5% respectively.

As under exogenous taxes, an increase in technology will increase domestic output, and despite the increased taxes, output increases by approximately the same amount. This is an interesting result, as according to the impulse responses with exogenous taxes, an increase in either tax rate ought to lower domestic output. Like before, the hours worked can be deduced from the production function, and as the technology shock is the same and output is approximately the same, we can deduce that households in country H will reduce their work effort approximately as before. In the presence of endogenous taxes, however, domestic households will actually reduce their consumption, exporting the windfall from the technology shock.

The terms of trade increase, indicating that households will switch consumption to goods produced in the home country. Noticing that foreign output rises without there having been a foreign technology shock lends itself to the conclusion that foreign households will increase hours worked. Foreign households increase hours worked to increase their income to con-



(a) All Variables



(b) Selected Variables

Figure 5: Impulse Response to a One-Percent Deviation in Domestic Technology – Nash Equilibrium for Endogenous Taxes, Rule (109)

sume both domestic and foreign goods. Note that there is PPI inflation in both countries instead of deflation as was the case under exogenous taxes, with the foreign PPI increasing much more than the domestic PPI. Despite the increase in the terms of trade, which would tend to decrease foreign PPI inflation, the combined effects of the increase in foreign consumption and output lead to a very high rate of foreign PPI inflation.

6.5 Fiscal Policy Interaction, Rule (110)

Table 5: Fiscal Policy Games, Specification (110)

	Nash	Best (H)	Best (F)	Best Coop
ϕ_τ^c	N.A.	-2.5	4.5	-0.5
ϕ_τ^y	N.A.	-1.5	0.5	-0.5
ϕ_τ^{c*}	N.A.	0.5	-2.5	-0.5
ϕ_τ^{y*}	N.A.	0.5	-2.5	-0.5
U	N.A.	-185.29	-187.61	-185.65
U^*	N.A.	-188.65	-187.47	-187.74

Table 5 summarizes the results of fiscal policy interaction using the feedback rule on output with the same range of potential parameter values as under the previous feedback rule. Disappointingly, I was unable to find a Nash equilibrium even after trying multiple potential parameter ranges. Particularly vexing is the result that the best possible cooperative solution has parameters taking the same values as under the previous feedback rule: if, under one-sided policy decisions, fiscal policy rules can generate welfare gains for both countries, why should the feedback parameters here take values as close to zero as possible? This makes me very suspicious of the results regarding utility in the table above, showing incredible utility gains to both countries (approximately 3.7% and 2.8% over the baseline measure for country H and F respectively). Yet, as the impulse response presented hereafter will show, these results may not be as implausible as first suspected. As such, I conclude that if cooperation is effected, weakly countercyclical taxes using output as feedback can induce significant gains to utility in both countries.

Impulse Responses to a One-Percent Deviation in Domestic Technology (a) and Government Expenditures (b) under Cooperation

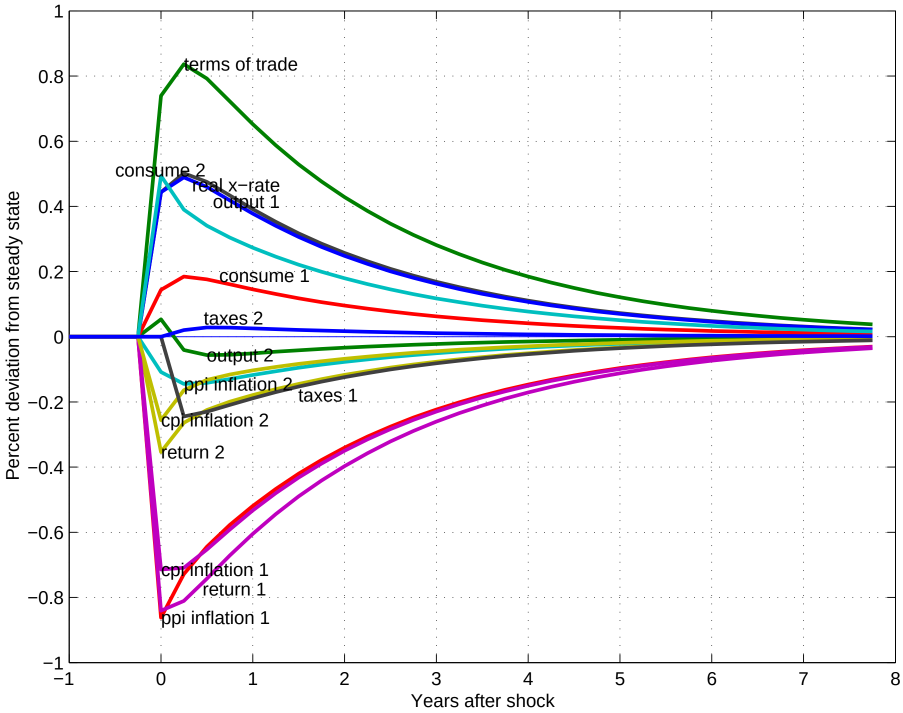
Figure 6 shows the impulse responses of selected variables to a positive shock in domestic technology (Figure 6(a)) and to a positive shock in domestic government expenditures (Figure 6(b)). The nominal exchange rate is omitted as it does not behave qualitatively differently than with exogenous taxes: the response is slightly amplified compared to the baseline model, but otherwise nothing changes. Notice that only "taxes" are included: as both consumption and production taxes in each country follow the same feedback rule, the deviations in both taxes in each country will be identical.

Examining Figure 6(a), notice that following the first quarter (taxes react with a lag here per assumption as mentioned previously), taxes in country F will increase due to the expectation in the previous period of decreased output and taxes in country H will deviate from their respective steady-state levels by about two-tenths of a percentage point.

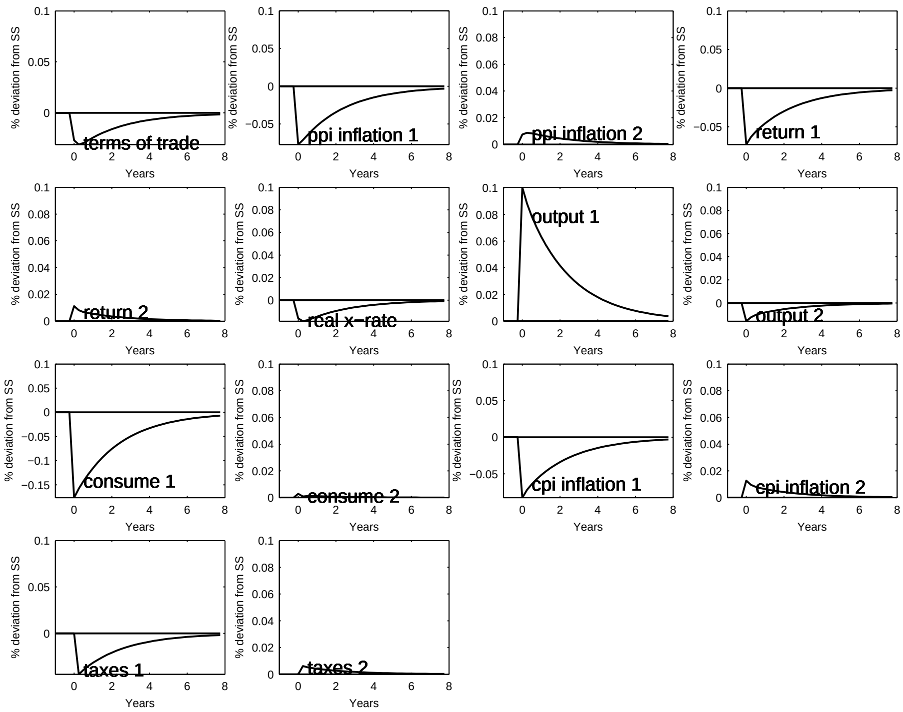
The combined effects of the increase in foreign taxes and the greater decline of domestic taxes would lend itself to the conclusion that domestic consumption should rise and foreign consumption fall, yet the opposite happens. As the real exchange rate is now higher, the marginal utility of consumption in country F is higher than that in country H, thus consumption in country F will increase and that in country H decrease to equalize. This can be seen most easily by noting that taxes are unchanged initially due to their lag, simplifying the intuition behind (50).

It is certainly noteworthy that foreign output, excepting for its initial response, deviates negatively from its steady state, more in line with the literature in open economy NKM's, e.g. Pappa (2002, p. 30). This could be considered an efficiency gain, as now the relative gain in production efficiency through a positive shock to technology in one country is reflected in a shift of production to the more efficiently producing country. The end result: households in both countries enjoy higher consumption and suffer less disutility from hours worked and , furthermore, households in country H work and consume relatively less than households in country F.

Turning to Figure 6(b), only sub-plots are given as the combined plot is illegible, the most promising result is that foreign consumption only deviates



(a) Technology



(b) Government Expenditures

Figure 6: Impulse Response to a One-Percent Deviation in Domestic Technology and Government Expenditures – Cooperative Equilibrium for Endogenous Taxes, Rule (110)

from its steady state initially. Once tax policies are initiated, consumption in country F is shielded from the demand shock specific to country H. Domestic consumption, however, is more starkly affected by the increase in government expenditures than was the case with exogenous taxes. As domestic households consume foreign goods as well as domestic ones, their demand for foreign goods falls in response to their reduction in overall consumption. Households in country F work less with consumption more or less unchanged: a boost to their utility. Effectively, the effect of a positive shock to domestic government consumption compared to that with exogenous taxes is that significantly more of the impact is shifted to country H, the location of the demand shock, reducing the effects on international transmission mechanisms (i.e. the terms of trade and the real exchange rate). Quote Uhlig (2002, p. 18), "[i]deally, fiscal policy should respond to the country-specific 'fiscal demand' shocks;" here, cooperative fiscal policy with both fiscal authorities responding in concert *makes* a government expenditure shock "country specific," by mitigating its international spill-over effects.

6.6 Monetary Policy

Ideally, I should determine whether there exist any Nash equilibria amongst the four policy makers, that is, I should find that combination of parameter values, from which no player would wish to deviate. Unfortunately, with my current computing power and the programs I have written, I should need decades (millennia including smoothing parameters), not to mention a rather substantial RAM upgrade, to generate the necessary payoff matrices. Furthermore, MATLAB seems to have trouble manipulating three and four dimensional matricies, though his problem could be overcome by using multiple two dimensional payoff matrices for each authority. As such, I am forced compromise and not examine Nash games amongst all authorities. Schmitt-Grohé and Uribe (2004), Schmitt-Grohé and Uribe (2005), and Siu (2004) advocate a Ramsey approach to a closed-economy model with nominal rigidities, allowing for a social planner to solve for the optimal path of the economy. As, however, Uhlig (2002) has pointed out, there are obvious political-economic reasons why societies have opted for a separation of monetary and fiscal policies, making the examination of cooperation between a fiscal and a monetary authority irrelevant. Nevertheless, that monetary authorities are not allowed to set policy and are instead simply given a Taylor Rule to follow mechanically is an undesirable, albeit necessary, assumption in my model.

6.7 Simulation Results

Using the Toolkit of Uhlig (1999), the moments of the models can be compared to those reported in empirical findings. Tables 6, 8, and 10 summarize the results from my models, showing the standard deviations of the variables and their temporal cross-correlations with domestic output at time t for the baseline model, the Nash equilibrium under the policy rule (109), and the best cooperative solution under the policy rule (110), where the HP-filter has been used with the quarterly calibration of my models.

The results for the volatilities of variables are rather disappointing: variables show a much higher standard deviation that reported in empirical findings. For example, Bachus, Kehoe, and Kydland (1992, p. 750) report a standard deviation of output in the United States of 1.71% using HP-filtered data from 1954 till 1989, whereas my model delivers a standard deviation of 7.84%. Though monetary policy in both models focuses on CPI inflation instead of PPI inflation or including the real exchange rate, I believe that volatilities of government purchases and taxes (all of which were calibrated to a volatility much greater than that of technology, see Table 1), are the main culprits of this excessive volatility. Under the Nash equilibrium with technology-feedback taxes, this issue is only made worse, as everything, with the notable exception of the nominal exchange rate, becomes more volatile. This suspicion is not wholly confirmed by the variances in the cooperative solution with output feedback: the variances of taxes are significantly reduced, some variables show reduced variances compared to the baseline model (e.g. output and consumption), while other are made more volatile (PPI and CPI inflation and the nominal interst rates). That the variances in all my models are off by a factor of five or so is very perplexing.

The cross-correlations of variable, however, provide some hope. Under exogenous taxes (Table 6), consumption in country H is highly correlated with contemporaneous output in country H and to exactly the same degree as reported by Bachus, Kehoe, and Kydland (1992, p. 750). Output is highly auto-correlative, but to a slightly lesser degree than reported by the afore-

Table 6: Standard Deviations and Correlations, Exogenous Taxes
Correlations with Output 1 at time t

	Std. Dev.	t-5	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4	t+5
Terms of Trade	0.1189	-0.03	0.05	0.16	0.29	0.44	0.55	0.42	0.27	0.15	0.04	-0.04
Nominal Exchange Rate	0.9528	-0.02	0.08	0.20	0.34	0.51	0.64	0.44	0.27	0.13	0.02	-0.06
PPI Inflation 1	0.0915	0.04	-0.08	-0.25	-0.45	-0.70	-0.95	-0.62	-0.37	-0.18	-0.03	0.08
PPI Inflation 2	0.0915	0.03	0.02	0.01	-0.02	-0.06	-0.16	-0.14	-0.10	-0.06	-0.03	-0.012
Return 1	0.0934	0.05	-0.07	-0.22	-0.42	-0.66	-0.92	-0.65	-0.41	-0.21	-0.06	0.06
Return 2	0.0934	0.02	-0.00	-0.03	-0.07	-0.13	-0.24	-0.14	-0.08	-0.04	-0.01	0.01
Real Exchange Rate	0.0713	-0.03	0.05	0.16	0.29	0.44	0.55	0.42	0.27	0.15	0.04	-0.04
Output 1	0.0784	-0.06	0.07	0.24	0.45	0.72	1.00	0.72	0.45	0.24	0.07	-0.06
Output 2	0.0784	-0.01	-0.01	-0.00	0.01	0.03	0.10	0.03	0.01	-0.00	-0.01	-0.01
Consume 1	0.0878	-0.05	0.04	0.16	0.32	0.52	0.76	0.53	0.33	0.17	0.04	-0.05
Consume 2	0.0878	-0.02	0.02	0.09	0.17	0.27	0.40	0.26	0.16	0.08	0.02	-0.03
CPI Inflation 1	0.0876	0.06	-0.07	-0.23	-0.44	-0.69	-0.96	-0.68	-0.43	-0.22	-0.06	0.06
CPI Inflation 2	0.0876	0.02	0.00	-0.02	-0.05	-0.10	-0.20	-0.11	-0.06	-0.03	-0.01	0.01
Technology 1	0.1183	-0.03	0.08	0.22	0.39	0.60	0.79	0.54	0.34	0.17	0.04	-0.05
Technology 2	0.1183	-0.02	-0.02	-0.02	-0.03	-0.02	0.04	0.04	0.03	0.02	0.01	0.01
Government 1	0.1570	-0.02	0.01	0.04	0.08	0.13	0.20	0.14	0.09	0.05	0.02	-0.01
Government 2	0.1570	0.01	0.01	0.01	0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.00	-0.00
Consumption Tax 1	0.1476	0.02	-0.00	-0.04	-0.08	-0.15	-0.24	-0.16	-0.10	-0.05	-0.01	0.02
Consumption Tax 2	0.1476	0.00	-0.01	-0.02	-0.03	-0.05	-0.05	-0.03	-0.02	-0.01	0.00	0.01
Production Tax 1	0.2790	0.04	-0.03	-0.12	-0.23	-0.36	-0.50	-0.33	-0.19	-0.09	-0.01	0.05
Production Tax 2	0.2790	0.01	0.01	0.02	0.02	0.02	-0.02	-0.02	-0.01	-0.01	-0.01	-0.01

mentioned authors. CPI inflation and the nominal gross interest in country H are both almost perfectly negatively correlated with output, whereas Cooley and Quadrini (2004, p. 193) report a correlation in the United States between 1959 and 1996 of positive 0.51 and 0.4 respectively. Canzoneri, Cumby, and Diba (2005, p. 40) note that developing models that overcoming this discrepancy is an important challenge to NKM's and report that the addition of rule-of-thumb consumers moves them in the right direction but fail to resolve this issue.

Table 7: International Correlation Comparison: Output and Consumption

	Exogenous Taxes	Nash Equil. (109)	Coop. (110)
Output	0.10	0.88	-0.10
Consumption	0.47	-0.36	0.55

On the international front, output in the two countries is almost completely non-correlated, this is due, in large part, to the fact that I assumed no correlation between the technology shocks of the two countries. Due to consumption risk sharing, both countries' consumption is positively correlated with output in country H, though the latter more strongly so than the former due to the assumed home bias and international elasticity of substitution. Bachus, Kehoe, and Kydland (1992, p. 752) note for a variety of countries that their output tends to be more highly correlated with American output than their consumption is with American consumption. This is a common shortcoming of open-economy NKM's throughout the literature.

Moving to technology-feedback taxes in the Nash equilibrium (Table 8), a number of dramatic changes can be seen. Domestic and foreign output is now highly correlated, without having assumed any correlation between the technology shocks in the respective countries. Table 7 compares the international correlations of output and consumption under the baseline model and under Nash endogenous taxes. Whereas as consumption was highly positively correlated across countries under exogenous taxes, it is negatively correlated when fiscal policy makers follow the rule I introduced. For the United States and Europe, Bachus, Kehoe, and Kydland (1992, p. 752) report values of 0.7 for output and 0.46 for consumption. Thus, what seemed like an improvement in my model (output more highly positively correlated internationally than consumption) is merely a trade of one puzzle for another, as consumption, despite risk sharing, becomes negatively correlated across countries. Though

Table 8: Standard Deviations and Correlations, Endogenous Taxes, Nash Equilibrium, Specification (109)

Correlations with Output 1 at time t

	Std. Dev.	t-5	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4	t+5
Terms of Trade	0.2592	0.02	0.01	-0.00	-0.03	-0.09	-0.10	-0.08	-0.06	-0.04	-0.02	-0.00
Nominal Exchange Rate	0.7069	-0.00	0.03	0.08	0.13	0.15	0.25	0.17	0.10	0.05	0.01	-0.02
PPI Inflation 1	0.1917	-0.04	0.06	0.19	0.37	0.61	0.76	0.52	0.32	0.16	0.04	-0.05
PPI Inflation 2	0.1754	-0.05	0.08	0.24	0.45	0.72	0.95	0.66	0.42	0.22	0.06	-0.06
Return 1	0.3131	-0.04	0.07	0.21	0.41	0.67	0.86	0.60	0.37	0.20	0.05	-0.05
Return 2	0.3146	-0.04	0.08	0.24	0.44	0.71	0.92	0.63	0.39	0.20	0.05	-0.06
Real Exchange Rate	0.1555	0.02	0.01	-0.00	-0.03	-0.09	-0.10	-0.08	-0.06	-0.04	-0.02	-0.00
Output 1	0.1077	-0.05	0.07	0.24	0.46	0.73	1.00	0.73	0.46	0.24	0.07	-0.05
Output 2	0.0954	-0.03	0.09	0.24	0.44	0.70	0.88	0.55	0.34	0.17	0.04	-0.07
Consume 1	0.2538	-0.05	0.03	0.13	0.27	0.47	0.63	0.49	0.31	0.17	0.06	-0.02
Consume 2	0.1837	0.01	0.07	0.15	0.24	0.34	0.44	0.23	0.13	0.05	-0.01	-0.05
CPI Inflation 1	0.1808	-0.04	0.06	0.20	0.38	0.63	0.81	0.55	0.35	0.18	0.05	-0.05
CPI Inflation 2	0.1809	-0.04	0.08	0.23	0.44	0.71	0.92	0.64	0.40	0.20	0.05	-0.06
Technology 1	0.1183	-0.02	0.07	0.18	0.32	0.47	0.67	0.46	0.29	0.15	0.04	-0.04
Technology 2	0.1183	-0.04	0.05	0.18	0.34	0.57	0.72	0.49	0.31	0.16	0.04	-0.05
Government 1	0.1570	-0.01	0.01	0.03	0.06	0.09	0.15	0.10	0.07	0.04	0.01	-0.01
Government 2	0.1570	0.00	0.00	0.01	0.01	0.00	-0.01	-0.01	-0.01	-0.00	-0.00	-0.00
Consumption Tax 1	0.1597	-0.09	-0.02	0.07	0.18	0.32	0.47	0.67	0.46	0.29	0.15	0.04
Consumption Tax 2	0.3727	-0.10	-0.04	0.05	0.18	0.34	0.57	0.72	0.49	0.31	0.16	0.04
Production Tax 1	0.0532	-0.09	-0.02	0.07	0.18	0.32	0.47	0.67	0.46	0.29	0.15	0.04
Production Tax 2	0.1597	-0.10	-0.04	0.05	0.18	0.34	0.57	0.72	0.49	0.31	0.16	0.04

Table 9: Correlation Comparison: Consumption and Government Expend.

Exogenous Taxes	Nash Equil. (109)	Coop. (110)
-0.50	-0.06	-0.42

the aforesaid authors did give one example where consumption is negatively correlated across countries (Australia/USA), output is also negatively correlated in this example; thus, the Nash equilibrium disconnects consumption "too much." Note that both CPI inflation and the interest rate have become positively correlated with output, being now more positively correlated with output than reported by Cooley and Quadrini (2004, p. 193).

With cooperative output-feedback taxes (Table 10), a few notable changes compared with the baseline model can be seen, though qualitatively, little changes. The nominal exchange rate is now more highly correlated with output than in the baseline model: any hope of disconnect is thusly dashed. Consumption in country F is now more highly correlated with output in country H than is consumption on country H, this is not surprising recalling the impulse responses to technology shown earlier, but not in line with the empirics. As shown again in Table 7, output in the two countries in now negatively correlated, moving the baseline model further away from observations; consumption in the countries is now more highly correlated than in either of the other models. There seems to be a trade-off through all three models: the higher the correlation between output across countries, the lower the correlation between consumption.

In Table 9, I show the correlations of government expenditures and consumption in country H. With exogenous taxes, the two are negatively correlated, highlighting the crowding-out effect that Canzoneri, Cumby, and Diba (2005) noted as being inconsistent with VAR-analyses. Interestingly, with taxes garnering feedback solely from past expectations of current technology, this correlation almost completely evaporates. This is strange as with taxes being only dependent upon technology, the impulse responses to a shock in government expenditures remains unchanged from the baseline specification. Furthermore, with output-feedback taxes, this negative correlation is reduced (i.e. less negatively correlated) despite the fact that government expenditures were borne almost exclusively by consumption.

Table 10: Standard Deviations and Correlations, Endogenous Taxes Cooperative Equilibrium, Specification (110)

Correlations with Output 1 at time t

	Std. Dev.	t-5	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4	t+5
Terms of Trade	0.1498	-0.01	0.10	0.24	0.41	0.60	0.70	0.53	0.35	0.18	0.05	-0.05
Nominal Exchange Rate	1.0863	0.00	0.12	0.27	0.44	0.63	0.71	0.48	0.29	0.14	0.02	-0.07
PPI Inflation 1	0.1001	0.01	-0.12	-0.29	-0.50	-0.74	-0.94	-0.61	-0.37	-0.18	-0.03	0.08
PPI Inflation 2	0.1007	0.04	0.04	0.03	0.02	-0.02	-0.15	-0.14	-0.11	-0.07	-0.05	-0.02
Return 1	0.1119	0.04	-0.09	-0.25	-0.45	-0.69	-0.92	-0.66	-0.42	-0.22	-0.07	0.05
Return 2	0.1122	0.02	0.00	-0.03	-0.07	-0.14	-0.27	-0.17	-0.10	-0.05	-0.02	0.01
Real Exchange Rate	0.0899	-0.01	0.10	0.24	0.41	0.60	0.70	0.53	0.35	0.18	0.05	-0.05
Output 1	0.0643	-0.05	0.09	0.27	0.50	0.76	1.00	0.76	0.50	0.27	0.09	-0.05
Output 2	0.0654	0.00	-0.03	-0.07	-0.11	-0.14	-0.10	-0.14	-0.11	-0.06	-0.03	0.00
Consume 1	0.0661	-0.04	-0.03	-0.00	0.03	0.09	0.23	0.21	0.15	0.10	0.05	0.02
Consume 2	0.0646	-0.01	0.10	0.23	0.40	0.59	0.77	0.48	0.28	0.13	0.02	-0.07
CPI Inflation 1	0.0954	0.04	-0.09	-0.26	-0.47	-0.72	-0.95	-0.69	-0.44	-0.24	-0.07	0.05
CPI Inflation 2	0.0960	0.02	0.01	-0.01	-0.03	-0.08	-0.19	-0.10	-0.06	-0.03	-0.01	0.01
Technology 1	0.1183	-0.02	0.12	0.30	0.51	0.76	0.95	0.65	0.40	0.20	0.04	-0.08
Technology 2	0.1183	-0.03	-0.05	-0.08	-0.10	-0.11	-0.02	-0.00	0.01	0.02	0.02	0.03
Government 1	0.1570	-0.01	0.01	0.05	0.10	0.16	0.24	0.17	0.11	0.06	0.02	-0.01
Government 2	0.1570	0.00	0.00	-0.00	-0.01	-0.02	-0.03	-0.02	-0.01	-0.01	-0.00	0.00
Consumption Tax 1	0.0308	0.13	0.02	-0.12	-0.30	-0.53	-0.78	-0.98	-0.69	-0.43	-0.22	-0.05
Consumption Tax 2	0.0314	-0.01	0.02	0.06	0.11	0.16	0.20	0.14	0.11	0.06	0.02	-0.01
Production Tax 1	0.0308	0.13	0.02	-0.12	-0.30	-0.53	-0.78	-0.98	-0.69	-0.43	-0.22	-0.05
Production Tax 2	0.0314	-0.01	0.02	0.06	0.11	0.16	0.20	0.14	0.11	0.06	0.02	-0.01

7 Discussion

The introduction of distortionary taxes into a New Keynesian open-economy model adds some very interesting and useful dynamics to the model. Consumption taxes especially have been shown to be particularly potent, driving a wedge in the consumption risk sharing obtained through uncovered interest rate parity. The baseline model fails to match international real business cycle facts on several levels, however, with standard deviations of most variables being five-fold what is observed in the data. Monetary policy based on a simple Taylor Rule does not deliver the dynamics that one would expect, temporarily increasing domestic output above its steady steady in response to a foreign technology shock.

With the policy rules I examined here, international fiscal policy cooperation is to be preferred to unilateral policy making. In the case of the technology shocks as feedback, random walk taxation appears to be superior to active taxation. Using output as feedback, however, makes both countries better off, assuming cooperation is achieved; it is certainly disappointing that I could not find a Nash equilibrium for this policy rule. Despite the lag in policy implementation, the latter rule is able to minimize the international spill over effects from a country-specific demand shock and to make some improvements over the impulse responses to a country-specific technology shock.

A multitude of extensions could be made, from adding wage rigidities along the lines of Erceg, Henderson, and Levin (2000) to the addition of capital with adjustment costs, e.g. Schmitt-Grohé and Uribe (2005). Both would add additional rigidities and the spectrum of taxes could be increased to include wage-income and capital taxes. Introducing a demand for money, either by postulating that households derive utility from holding real balances, e.g. Obstfeld and Rogoff (1994), or by cash-in-advance constraints to consumer purchases could prove particularly insightful with modifications to the government budget constraint, allowing the investigation of the Freidman rule (the satiation of "the economy with real balances by generating a deflation that drives the net nominal interest rate to zero" (Ljungqvist and Sargent 2004, p. 880)).

Personally, I am inclined to conclude that any extensions would be premature before some technical issues are dealt with satisfactorily. The measure of welfare used here could be misleading to such an extent that policy decisions based on it may be sub-optimal. Furthermore, that technical limitations restricted the number of simulations that could be used in generating payoff matrices has the potential of creating equilibria that are specific to the particular draws of exogenous factors. Any thought to extensions should come second to overcoming these technical problems.

It would certainly be desirable to studying potential Nash equilibria when both fiscal and monetary policy makers are allowed to determine their policies contemporaneously instead of limiting monetary policy makers to a simple Taylor Rule as was done here. There are many potential monetary policy rules to choose from and forward-looking rules and those designed more specifically for use in open economies could be compared. Fiscal policy could also be expanded to provide more realistic insights into the trade-offs that fiscal authorities face: debt dynamics and endogenous government expenditures would confront fiscal authorities with intertemporal and demand-modifying problems. Once again though, correcting for the problems raised in the foregoing paragraph should be paramount to this.

8 Summary and Concluding Remarks

I have derived a two-country model with staggered price-setting and monopolistic competition to examine the effects of distortionary taxation in the Open Economy New Keynesian setting. With international risk sharing being a crucial link between these economies, I have shown the fiscal authorities can use a consumption tax to manipulate this interdependency. With the welfare measure and simulation techniques I have used here, cooperative policy using technology-feedback rules has been shown to be preferable to unilateral policy setting. Interestingly, though not all too surprising in light of the literature and the known general sub-optimality of distortionary taxes to lump-sum taxes, the cooperative solution with technology as feedback calls for setting taxes close to a random walk, though marginally negative parameters may induce a slight welfare gain. Using output as feedback, however, cooperation is able to generate significant welfare gains with a mildly active fiscal policy.

9 References

References

Bachus, D. K., P. J. Kehoe, and F. E. Kydland (1992): "International Real Business Cycles," *The Journal of Political Economy*, 100(4).

Ball, L. (1998): "Policy Rules for Open Economies," Johns Hopkins University, Research Discussion Paper 9806.

Beetsma, R. M., and H. Jensen (2002): "Monetary and Fiscal Policy Interactions in a Micro-founded Model of a Monetary Union," *ECB Working Paper No. 166.*

Benigno, G., and P. Benigno (2002): "Price Stability in Open Economies," The London School of Economics and New York University.

——— (2004): "Optimal Monetary Policy in a Currency Area," *Journal of Internaional Economics*, 63, 293–320.

Benigno, P. (2001): "Price Stability with Imperfect Financial Integration," New York University and CEPR.

Calvo, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

Canzoneri, M. B., R. E. Cumby, and B. T. Diba (2004): "The Need for International Policy Coordination: What's Old, What's New, Whats Yet to Come?," Forthcoming, JIE.

——— (2005): "How Do Monetary and Fiscal Policy Interact in the European Monetary Union?," *NBER Working Paper No. 11055.*

Clarida, R., J. Galí, and M. Gertler (1998): "Monetary Policy Rules in Practice: Some International Evidence," *European Economic Review*, 42, 1033–1067.

——— (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37, 1661–1707.

——— (2002): "A Simple Framework for International Monetary Policy Analysis," *Journal of Monetary Economics*, 49, 879–904.

Cooley, T. F., and V. Quadrini (2004): "Optimal Monetary policy in a Phillips-Curve World," *Journal of Economic Theory*, 118, 174–208.

Corsetti, G., and P. Pesenti (2001): "Welfare and Macroeconomic Interdependence," *The Quarterly Journal of Economics*.

——— (2002): "Self-Validating Optimum Currency Areas," *NBER Working Paper No. 8783.*

——— (2005): "International Dimensions of Optimal Monetary Policy," *Journal of Monetary Economics*, 52, 281–305.

Cukierman, A. (2005): "Keynesian Economics, Monetary Policy and the Business Cycle - New and Old," Berglas School of Economics, Tel-Aviv University and CEPR.

Devereux, M. B., and C. Engel (2000): "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility," *NBER Working Paper No. 7665.*

——— (2002): "Exchange Rate Pass-Through, Exchange Rate Volatility, and Exchange Rate Disconnect," *NBER Working Paper No. 8858.*

Dixit, A. K., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297–308.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46, 281–313.

Galí, J., and T. Monacelli (2002): "Monetary Policy and Exchange Rate Volatility in a Small Open Eonomy," *NBER Working Paper No. 8905.*

——— (2004): "Optimal Fiscal Policy in a Monetary Union," CREI, Universitat Pompeu Fabra, IGIER, Universitá Bocconi, and CEPR.

Herz, B., W. Roeger, and L. Vogel (2004): "Fiscal Stabilization in a Small Open Economy," .

Kim, J., and S. H. Kim (2003): "Spurious Welfare Reversals in International Business Cycle Models," *Journal of International Economics*, 60, 471–500.

——— (2004): "Welfare Effects of Tax Policy in Open Economies: Stabilization and Cooperation," Federal Reserve Board and Tufts University.

Kydland, F. E., and E. C. Prescott (1977): "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *The Journal of Polictical Economy*, 85(3), 473–492.

Lane, P. R. (2001): "The New Open Economy Macroeconomics: A Survey," *Journal of International Economics*, 54, 235–266.

Ljungqvist, L., and T. J. Sargent (2004): *Recursive Macroeconomic Theory*. The MIT Press, second edn.

Lombardo, G., and A. Sutherland (2003): "Monetary and Fiscal Interactions in Open Economies," *ECB Working Paper No. 289*.

Obstfeld, M., and K. Rogoff (1994): "Exchange Rate Dynamics Redux," *NBER Working Paper No. 4693*.

——— (2000): "Do We Really Need a New International Monetary Compact?," *NBER Working Paper No. 7864*.

Pappa, E. (2002): "Do the ECB and the Fed Really Need to Cooperate? Optimal Monetary Policy in a Two-Country World," Department of Economics, The London School of Economics and Political Science.

Romer, D. (2001): *Advanced Macroeconomics*. McGraw-Hill, second edn.

Schmitt-Grohé, S., and M. Uribe (2004): "Optimal Fiscal and Monetary Policy under Sticky Prices," *Journal of Economic Theory*, 114, 198–230.

——— (2005): "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model," CEPR, NBER, and Duke University.

Siu, H. E. (2004): "Optimal Fiscal and Monetary Policy with Sticky Prices," *Journal of Monetary Economics*, 51, 575–607.

Taylor, J. B. (1993): "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Serires on Public Policy*, 39, 195–214.

Tille, C. (1999): "The Role of Consumption Substitutability in the International Transmission of Shocks," Federal Reserve Bank of New York.

Uhlig, H. (1999): "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily," in *Computational Methods for the Study of Dynamic Economies*, ed. by R. Marimon, and A. Scott, pp. 30–61. Oxford University Press, Oxford.

——— (2002): "One Money, but Many Fiscal Policies in Europe: What are the Consequences?," Humboldt University Berlin and CEPR.

Woodford, M. (2001): "Fiscal Requirements for Price Stability," *NBER Working Paper No. 8072*.

Yun, T. (2004): "Optimal Monetary Policy with Relative Price Distortions," Kookmin University and The University of Southern California.

A Appendices

A.1 Optimal Consumption Allocation

Households, in determining the allocation of their consumption, minimize the cost of consumption given an aggregation technology. Following the timing found in the main text, in determining its allotment to all individual goods produced in country H, any given household, and since they all have identical preferences and aggregating technologies the representative household, wants to minimize the cost of its domestic consumption given its domestic consumption and a technology to assemble it. Thus,

$$\min_{C_{H,t}(j)} \int_0^1 P_{H,t}(j) C_{H,t}(j) dj \equiv P_{H,t} C_{H,t}$$

s.t.

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{111}$$

Forming the Lagrangian,

$$\mathcal{L} = -\int_0^1 P_{H,t}(j) C_{H,t}(j) dj - \lambda_t \left(-C_{H,t} - \left(\int_0^1 C_{H,t}(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \right) \tag{112}$$

and taking the derivative yields,

$$\frac{\partial \mathcal{L}}{\partial C_{H,t}(j)} \stackrel{!}{=} 0 = -P_{H,t}(j) + \frac{\eta}{\eta-1} \lambda_t \left(\int_0^1 C_{H,t}(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{1}{\eta-1}} \frac{\eta-1}{\eta} C_{H,t}(j)^{\frac{-1}{\eta}} \tag{113}$$

which, after simplifying and having used the definition of $C_{H,t}$, produces,

$$P_{H,t}(j) = \lambda_t {C_{H,t}}^{\frac{1}{\eta}} C_{H,t}(j)^{\frac{-1}{\eta}} \tag{114}$$

Solving for $C_{H,t}(j)$ yields,

$$C_{H,t}(j) = {\lambda_t}^{\eta} P_{H,t}(j)^{-\eta} C_{H,t} \tag{115}$$

Inserting this into the definition of the bundle, found in (111) as the restriction to the minimization problem, produces,

$$C_{H,t} = \left(\int_0^1 \left({\lambda_t}^{\eta} P_{H,t}(j)^{-\eta} C_{H,t} \right)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{116}$$

which, after simplifying and having solved for λ_t, yields,

$$\lambda_t = \left(\int_0^1 P_{H,t}(j)^{1-\eta} dj \right)^{\frac{1}{1-\eta}} \tag{117}$$

Substituting this into (114) yields the following equality for $P_{H,t}(j)$,

$$P_{H,t}(j) = \left(\int_0^1 P_{H,t}(j)^{1-\eta} di\right)^{\frac{1}{1-\eta}} C_{H,t}{}^{\frac{1}{\eta}} C_{H,t}(j)^{\frac{-1}{\eta}} \tag{118}$$

Now, this equality is substituted into the definition of $P_{H,t}C_{H,t}$, found as the minimization problem facing the bundler in (111), producing,

$$P_{H,t}C_{H,t} = \int_0^1 \left(\int_0^1 P_{H,t}(j)^{1-\eta} di\right)^{\frac{1}{1-\eta}} C_{H,t}{}^{\frac{1}{\eta}} C_{H,t}(j)^{\frac{-1}{\eta}} C_{H,t}(j) dj \tag{119}$$

After simplifying and having used the definition of $C_{H,t}$, the restriction to the minimization problem (111), the price for the bundle is obtained as,

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\eta} dj\right)^{\frac{1}{1-\eta}} \tag{120}$$

corresponding to (7) in the main text. Furthermore, substituting this into (118) and solving for $C_{H,t}(j)$ delivers,

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\eta} C_{H,t} \tag{121}$$

corresponding to (8) in the main text.

The household, while alloting $C_{F,t}$, faces an analogous problem, which delivers expressions for $P_{F,t}$ and $C_{F,t}(j)$, (9) and (10) in the main text, using the definition of $C_{F,t}$, given as (6) also in the main text.

The problem for households assembling the aggregate consumption bundle is similar. The cost minimization problem is,

$$min_{C_{H,t},C_{F,t}} P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \equiv P_t C_t$$

s.t.

$$C_t = \left[(1-\theta)^{\frac{1}{\omega}} C_{H,t}{}^{\frac{\omega-1}{\omega}} + \theta^{\frac{1}{\omega}} C_{F,t}{}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{122}$$

Likewise analogous are the problems for foreign households.

A.2 Steady State

Assumptions

$$\bar{s} = 1, \ \bar{e} = 1 \ \bar{h} = \bar{h^*} = 1, \ \bar{Q} = \bar{Q^*} = 0 \tag{123}$$

From (44),

$$\bar{R} = \frac{1}{\beta} \tag{124}$$

From (30) and assumptions,

$$\bar{p_H} = \bar{p_F} \tag{125}$$

From the foregoing and (11),

$$\bar{p} = \bar{p_H} = \bar{p_F} \tag{126}$$

From (126) and (12),

$$\bar{c_H} = (1-\theta)\bar{c} \tag{127}$$

From (126) and (13),

$$\bar{c_F} = \theta\bar{c} \tag{128}$$

From (126) and (18),

$$\bar{c_F^*} = (1-\theta)\bar{c^*} \tag{129}$$

From (126) and the second equation in (18),

$$\bar{c_H^*} = \theta\bar{c^*} \tag{130}$$

From (53),

$$ln(\bar{z}) = \rho_z ln(\bar{z}) \Rightarrow \bar{z} = 1 \tag{131}$$

From (52),

$$\bar{y} = \bar{z}\bar{h}^{\gamma} = 1 \tag{132}$$

From (54),

$$ln(\bar{z}^*) = \rho_{z^*} ln(\bar{z}^*) \Rightarrow \bar{z}^* = 1 \tag{133}$$

From the foreign production function,

$$\bar{y}^* = \bar{z}^*\bar{h}^{*\gamma} = 1 \tag{134}$$

From (61)

$$\bar{p_H} = (1-\alpha)\bar{\tilde{p}_H} + \alpha\bar{p_h} \Rightarrow \bar{p_h} = \bar{\tilde{p}_H} \tag{135}$$

Note that in the steady state, all domestic prices are equal (the same holds for foreign prices).

$$\bar{p}^* = \bar{p_F}^* = \bar{(p_H)}^* = \bar{\tilde{p}_F}^* = \frac{\bar{p}}{\bar{e}} = \bar{p} \tag{136}$$

Normalizing $\bar{p}$ to one, the world resource (74) can be written as,

$$2 = \bar{c} + \bar{c^*} + \bar{g} + \bar{g^*} \tag{137}$$

From (76),

$$\bar{d} = (1 - \alpha) + \alpha \bar{d} \Rightarrow \bar{d} = 1 \tag{138}$$

and thus (75),

$$1 = (1 - \theta)\bar{c} + \theta \bar{c^*} + \bar{g} \tag{139}$$

Subtracting this from (137) gives,

$$1 = (1 - \theta)\bar{c^*} + \theta \bar{c} + \bar{g^*} \tag{140}$$

Combining the previous two yields

$$(1 - 2\theta)\bar{c} + \bar{g} = (1 - 2\theta)\bar{c}^* + \bar{g}^* \tag{141}$$

which, with (33) and (66),

$$1 = \bar{g} + \bar{c} \tag{142}$$

and its foreign analog,

$$1 = \bar{g}^* + \bar{c}^* \tag{143}$$

delivers,

$$\bar{c} = \bar{c}^* \tag{144}$$

and,

$$\bar{g} = \bar{g}^* \tag{145}$$

Combining (37),

$$\bar{\lambda} = \frac{1}{\bar{c}^{\sigma_c} \bar{p} \left(1 + \bar{\tau^c}\right)} \tag{146}$$

with (58), (59), and (60),

$$\begin{aligned}
\bar{\tilde{P}}_H &= \frac{\eta}{\eta - 1} \frac{\bar{x_1}}{\bar{x_2}} \\
\bar{x_1} &= \frac{\kappa_H}{\gamma} \frac{1}{1 - \alpha\beta} \\
\bar{x_2} &= \frac{\lambda \left(1 - \bar{\tau^y}\right)}{1 - \alpha\beta}
\end{aligned} \tag{147}$$

yields,

$$1 = \frac{\eta}{\eta - 1} \frac{\kappa_H}{\gamma} \bar{c}^{\sigma_c} \frac{\left(1 + \bar{\tau^c}\right)}{\left(1 - \bar{\tau^y}\right)} \tag{148}$$

Solving for $\bar{\tau^y}$ delivers (80) in the main text,

$$\bar{\tau}^y = 1 - \bar{c}^{\sigma_c} \frac{\gamma}{\kappa_h} \frac{\eta - 1}{\eta} (1 + \bar{\tau}^c) \tag{149}$$

with its foreign analog, yielded by the foreign equivalents to the foregoing five equation, being,

$$\bar{\tau}^{y*} = 1 - \bar{c}^{*\,\sigma_c} \frac{\gamma}{\kappa_h} \frac{\eta - 1}{\eta} (1 + \bar{\tau}^{c*}) \tag{150}$$

Though not necessary for the system, note that (50) delivers

$$\Xi = \frac{(1 + \bar{\tau}^{c*})}{(1 + \bar{\tau}^c)} \tag{151}$$

Thus, under the assumptions made here, the initial conditions regarding the real exchange rate will manifest themselves in steady-state tax rates. If we assume $\Xi = 1$, as do Devereux and Engel (2002, p. 7), the steady-state tax rates on consumption must be equal, and thus, those on production as well.

A.3 Derivation of the Log-Linearized Model

From (4),

$$\hat{c}_t = (1 - \theta)\hat{c}_{H,t} + \theta \hat{c}_{F,t} \tag{152}$$

From the foreign analog (14),

$$\hat{c}^*_t = (1 - \theta)\hat{c}^*_{F,t} + \theta \hat{c}^*_{H,t} \tag{153}$$

From (11),

$$\hat{p}_t = (1 - \theta)\hat{p}_{H,t} + \theta \hat{p}_{F,t} \tag{154}$$

From the foreign analog (16),

$$\hat{p}^*_t = (1 - \theta)\hat{p}^*_{F,t} + \theta \hat{p}^*_{H,t} \tag{155}$$

From (12),

$$\hat{c}_{H,t} = \hat{c}_t - \omega \left(\hat{p}_{H,t} - \hat{p}_t\right) \tag{156}$$

From its foreign analog (18),

$$\hat{c}^*_{F,t} = \hat{c}^*_t - \omega \left(\hat{p}^*_{F,t} - \hat{p}^*_t\right) \tag{157}$$

From (28),

$$\hat{e}_t = \hat{p}_{H,t} - \hat{p}^*_{H,t} \tag{158}$$

And its analog for prices F (29),

$$\hat{e}_t = \hat{p}_{F,t} - \hat{p}^*_{F,t} \tag{159}$$

From (30),

$$\hat{s}_t = \hat{p}_{F,t} - \hat{p}_{H,t} \tag{160}$$

From (40),

$$\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1} \tag{161}$$

And its foreign analog (43),

$$\hat{\pi}^*_t = \hat{p}^*_t - \hat{p}^*_{t-1} \tag{162}$$

From (44),

$$0 = \sigma_c \hat{c}_t - \sigma_c E_t\left[\hat{c}_{t+1}\right] - E_t\left[\hat{\pi}_{t+1}\right] + \frac{\hat{\tau}^c_t}{1+\bar{\tau}^c} - \frac{E_t\left[\hat{\tau}^c_{t+1}\right]}{1+\bar{\tau}^c} + \hat{R}_t \tag{163}$$

which corresponds directly to (86). From its foreign analog (45),

$$0 = \sigma_c \hat{c}^*_t - \sigma_c E_t\left[\hat{c}^*_{t+1}\right] - E_t\left[\hat{\pi}^*_{t+1}\right] + \frac{\hat{\tau}^{c*}_t}{1+\bar{\tau}^{c*}} - \frac{E_t\left[\hat{\tau}^{c*}_{t+1}\right]}{1+\bar{\tau}^{c*}} + \hat{R}^*_t \tag{164}$$

which corresponds directly to (87). From (48),

$$0 = E_t\left[\hat{e}_{t+1}\right] - \hat{e}_t + \hat{R}^*_t - \hat{R}_t \tag{165}$$

which corresponds directly to (90). From (49),

$$\hat{q}_t = \hat{e}_t + \hat{p}^*_t - \hat{p}_t \tag{166}$$

From (58),

$$\hat{\tilde{p}}_{H,t} = \hat{x}_{1,t} - \hat{x}_{2,t} \tag{167}$$

From (59),

$$\hat{x}_{1,t} = (1-\alpha\beta)\frac{1+\sigma_h}{\gamma}\left[\hat{y}_t - \hat{z}_t\right] + \alpha\beta E_t\left[\hat{x}_{1,t+1}\right] \tag{168}$$

From (60),

$$\hat{x}_{2,t} = (1-\alpha\beta)\left[\hat{y}_t - \sigma_c \hat{c}_t - \hat{p}_t - \frac{\hat{\tau}^c_t}{1+\bar{\tau}^c} - \frac{\hat{\tau}^y_t}{1-\bar{\tau}^y}\right] + \alpha\beta E_t\left[\hat{x}_{2,t+1}\right] \tag{169}$$

From (61),

$$\hat{p}_{H,t} = (1-\alpha)\hat{\tilde{p}}_{H,t} + \alpha\hat{p}_{H,t-1} \tag{170}$$

The foreign analogs to the previous four equations are, respectively, from (62),

$$\hat{\tilde{p}}^*_{F,t} = \hat{x}^*_{1,t} - \hat{x}^*_{2,t} \tag{171}$$

from (63),

$$\hat{x}^*_{1,t} = (1-\alpha\beta)\,\frac{1+\sigma_h}{\gamma}\,[\hat{y}^*_t - \hat{z}^*_t] + \alpha\beta E_t\left[\hat{x}^*_{1,t+1}\right] \tag{172}$$

from (64),

$$\hat{x}^*_{2,t} = (1-\alpha\beta)\left[\hat{y}^*_t - \sigma_c\hat{c}^*_t - \hat{p}^*_t - \frac{\hat{\tau}^{c*}_t}{1+\bar{\tau}^{c*}} - \frac{\hat{\tau}^{y*}_t}{1-\bar{\tau}^{y*}}\right] + \alpha\beta E_t\left[\hat{x}^*_{2,t+1}\right] \tag{173}$$

and from (65),

$$\hat{p}^*_{F,t} = (1-\alpha)\,\hat{\tilde{p}}^*_{F,t} + \alpha\hat{p}^*_{F,t-1} \tag{174}$$

From (74),

$$\hat{y}_t + \hat{p}_{H,t} + \hat{y}^*_t + \hat{p}^*_{F,t} = \bar{c}\hat{p}_t + \bar{c}\hat{c}_t + \bar{c}\hat{p}^*_t + \bar{c}\hat{c}^*_t + \bar{g}\hat{p}_{H,t} + \bar{g}\hat{g}_t + \bar{g}\hat{p}^*_{F,t} + \bar{g}^*\hat{g}^*_t \tag{175}$$

From (75),

$$\hat{y}_t = (1-\theta)\bar{c}\hat{c}_{H,t} + \theta\bar{c}\hat{c}^*_{H,t} + \bar{g}\hat{g}_t + \hat{d}_t \tag{176}$$

From (76),

$$\hat{d}_t = \eta\left[\hat{p}_{H,t} - (1-\alpha)\hat{\tilde{p}}_{H,t} - \alpha\hat{p}_{H,t-1}\right] + \alpha\hat{d}_{t-1} \tag{177}$$

Starting backwards with (177), notice that in combination with (170), this reduces to

$$\hat{d}_t = \alpha\hat{d}_{t-1} \tag{178}$$

assume initial conditions $\hat{d}_{-1} = 0$, then

$$\hat{d}_t = 0,\ \forall\, t \tag{179}$$

Combining (153), (155), and (157) (or "cheating" and log-linearizing the second equation in (18)) yields,

$$\hat{c}^*_{H,t} = \hat{c}^*_t - \omega\left(\hat{p}^*_{H,t} - \hat{p}^*_t\right) \tag{180}$$

Combining (155), (158), (159), and (160),

$$\hat{p}^*_{H,t} - \hat{p}^*_t = -(1-\theta)\hat{s}_t \tag{181}$$

Combining (154) and (160),

$$\hat{p}_{H,t} - \hat{p}_t = -\theta \hat{s}_t \tag{182}$$

Using the four foregoing expressions with (156), (176) can be rewritten as

$$\hat{y}_t = (1-\theta)\bar{c}\hat{c}_t + \theta\bar{c}\hat{c}_t^* + 2\bar{c}(1-\theta)\omega\theta\hat{s}_t + \bar{g}\hat{g}_t \tag{183}$$

corresponding to (82) in the main text.

Using (154), (155), (158), (159), and 160),

$$\begin{aligned} \hat{p}_{H,t} + \hat{p}_{F,t}^* - \bar{c}\hat{p}_t - \bar{c}\hat{p}_t^* - \bar{g}\hat{p}_{H,t} - \bar{g}\hat{p}_{F,t}^* &= \\ \bar{c}\left(\hat{p}_{H,t} - \hat{p}_t\right) + \bar{c}\left(\hat{p}_{F,t}^* - \hat{p}_t^*\right) &= \\ \bar{c}\left(-\theta\hat{s}_t + \theta\hat{s}_t\right) = 0 \end{aligned} \tag{184}$$

Combining this with (175) gives,

$$0 = -\hat{y}_t - \hat{y}_t^* + \bar{c}\hat{c}_t^* + \bar{c}\hat{c}_t + \bar{g}\hat{g}_t^* + \bar{g}\hat{g}_t \tag{185}$$

corresponding to (83) in the main text.

Defining (log-linearized) Producer-Price-Index inflation for country H as,

$$\hat{\pi}_{H,t} = \hat{p}_{H,t} - \hat{p}_{H,t-1} \tag{186}$$

and for country F as,

$$\hat{\pi}_{H,t} = \hat{p}_{F,t}^* - \hat{p}_{F,t-1}^* \tag{187}$$

Thus, (170) and (174) can be rewritten as,

$$\hat{\pi}_{H,t} = (1-\alpha)\left[\hat{\tilde{p}}_{H,t} - \hat{p}_{H,t-1}\right] \tag{188}$$

and

$$\hat{\pi}_{F,t}^* = (1-\alpha)\left[\hat{\tilde{p}}_{F,t}^* - \hat{p}_{F,t-1}^*\right] \tag{189}$$

Following Galí and Monacelli (2002, pp. 29-30), one can combine (167), (168), and (169), subtract $\hat{p}_{H,t-1}$ from both sides, and rearrange to yield:

$$\begin{aligned} \hat{\tilde{p}}_{H,t} - \hat{p}_{H,t-1} &= (1-\alpha\beta)\left[\frac{1+\sigma_h-\gamma}{\gamma}\hat{y}_t - \frac{1+\sigma_h}{\gamma}\hat{z}_t\right. \\ &+ \left.\sigma_c\hat{c}_t + \hat{s}_t + \frac{\hat{\tau}_t^c}{1+\bar{\tau}^c} + \frac{\hat{\tau}_t^y}{1-\bar{\tau}^y}\right] \\ &+ p_{H,t} - \hat{p}_{H,t-1} + \alpha\beta E_t\left[\hat{\tilde{p}}_{H,t+1} - \hat{p}_{H,t}\right] \end{aligned} \tag{190}$$

which, combined with (188) delivers,

$$0 = -\hat{\pi}_{H,t} + \zeta_\pi \left[\zeta_y \hat{y}_t - \zeta_z \hat{z}_t + \sigma_c \hat{c}_t + \hat{s}_t + \frac{\hat{\tau}_t^c}{1+\bar{\tau}^c} + \frac{\hat{\tau}_t^y}{1-\bar{\tau}^y} \right] + \beta E_t \left[\hat{\pi}_{H,t+1} \right] \tag{191}$$

corresponding to (88) in the main text. Its foreign analog, (89) in the main text, can be derived similarly. (166) and (159), with (160), can be rewritten in expected differences, so as to correspond with (165), to yield (92) and (91) in the main text. Finally,(182) can be combined with its equivalent in the foregoing period to yield (84) in the main text, with (85) being derived analogously.

I, Alexander Warren Zieroth, do hereby attest that I authored this master's thesis independently and without sources other than those indicated herein. All passages taken, either literally or in general matter, from publications or other resources are marked as such.

Alexander Warren Zieroth
Berlin, August 5, 2005